AQA Qualifications

AQA Level 2 Certificate FURTHER MATHEMATICS

Level 2 (8365)

Miscellaneous Worksheet

Version 2.0

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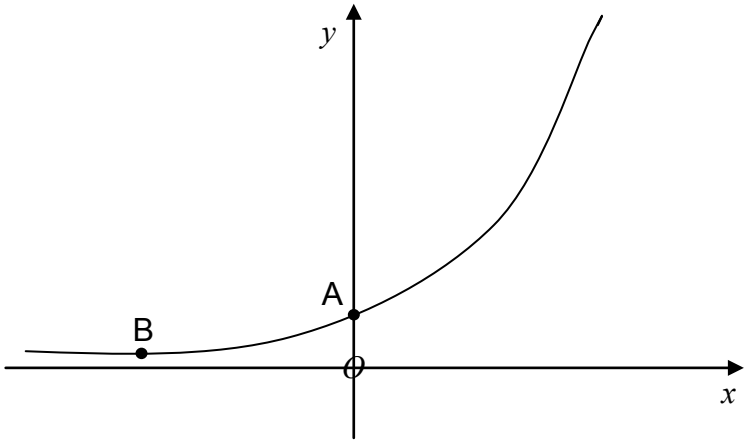
M Miscellaneous

Question 1 (Spec ref 2.13)

A (0, 8) and B (–3, 1) are points on $y = ab^x$ as shown.



By working out the values of a and b, show that the equation of the curve can be written in the form $y = 2^{x+3}$

(4 marks)

Question 2 (Spec ref 1.2)

Here are five cards.

2	3	4	5	6

Using four or five of the cards, how many numbers greater than 4000 can be made?

(4 marks)

Question 3 (Spec ref 2.9/2.20)

Two sequences S and T have nth terms

$$S_n = \frac{2n+3}{n} \quad \text{and} \quad T_n = \frac{30}{3n+4}$$

Use an algebraic method to work out the value of n when $S_n + T_n = 3$

(5 marks)

Question 4 (Spec Ref 2.18)

By expanding and simplifying, solve

$$\left(2x^{\frac{5}{2}} - x^{\frac{1}{2}}\right)^2 = x(1+4x^4)+108$$

(5 marks)

Question 5 (Spec ref 2.7)

In the expansion of $(a+5x)^4$ where $a > 0$

The coefficient of x is three times the coefficient of x^2.

Work out the value of a.

(5 marks)

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Worksheet 1

Coordinate Geometry Circles

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1 Coordinate Geometry – Circles

Question 1

Write down the equation of each of these circles.

(a) Centre (0, 3) radius 2 *(2 marks)*

(b) Centre (1, –5) radius 4 *(2 marks)*

(c) Centre (–3, 4) radius $\sqrt{7}$ *(2 marks)*

(d) Centre (8, 15) radius 17

Does this circle pass through the origin?

Show working to support your answer. *(4 marks)*

Question 2

Write down the centre and radius of each of these circles.

(a) $x^2 + y^2 = 36$ *(2 marks)*

(b) $(x - 3)^2 + (y - 4)^2 = 100$ *(2 marks)*

(c) $(x + 5)^2 + y^2 = 3$ *(2 marks)*

Question 3 (non-calculator)

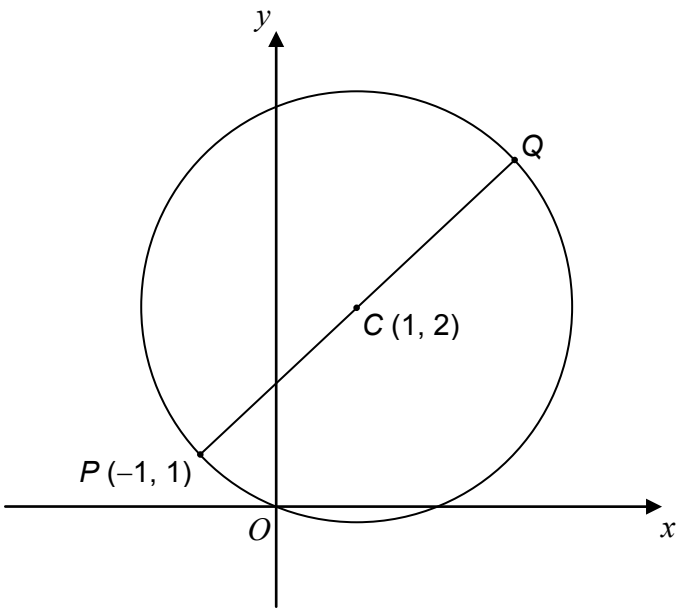
AB is the diameter of a circle.

A is (–3, 6) and *B* is (5, 12).

Work out the equation of the circle. *(5 marks)*

Question 4 (non-calculator)

PQ is a diameter of a circle, centre *C*.



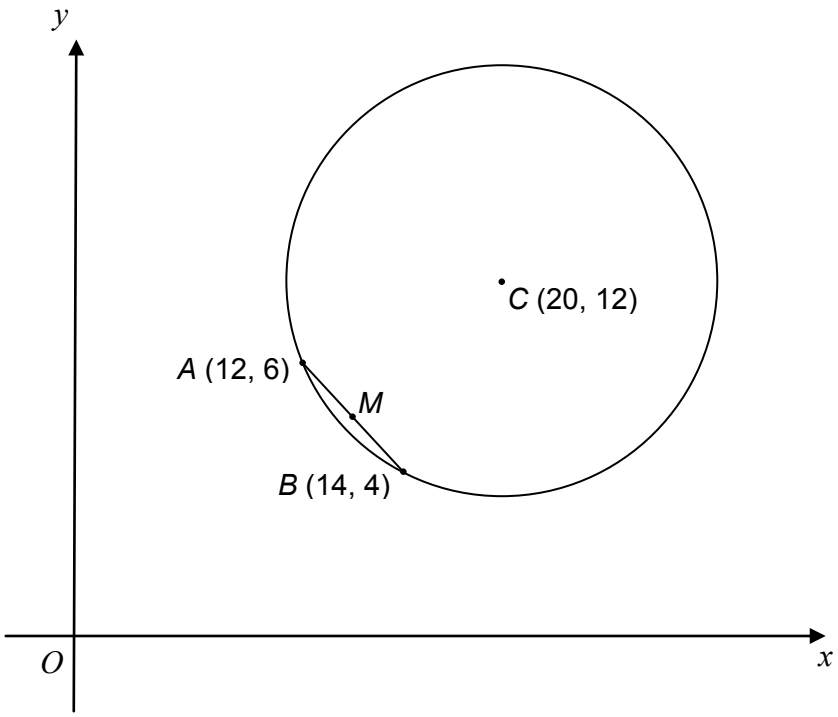
Not drawn accurately

(a) Work out the coordinates of *Q*. *(1 mark)*

(b) Work out the equation of the circle. *(3 marks)*

Question 5 (non-calculator)

A (12, 6) and *B* (14, 4) are two points on a circle, centre *C* (20, 12).



Not drawn accurately

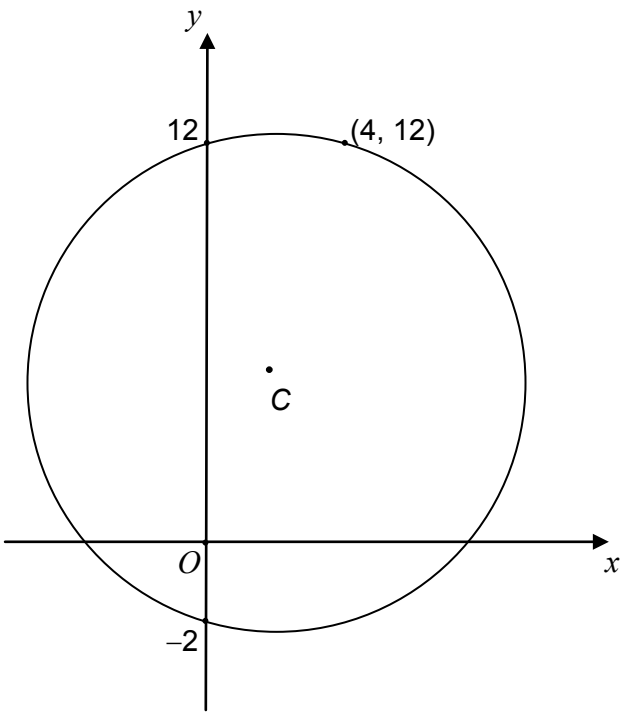
(a) Work out the coordinates of the midpoint *M*, of *AB*. *(2 marks)*

(b) Show that the length $CM = 7\sqrt{2}$ (*3 marks*)

(c) Work out the radius of the circle. *(2 marks)*

Question 6

(0, –2), (0, 12) and (4, 12) are three points on a circle, centre *C*.

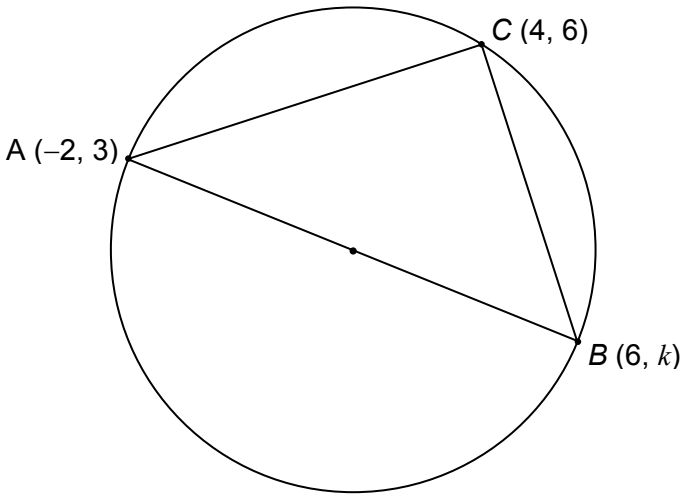


Not drawn accurately

Work out the coordinates of *C*. *(3 marks)*

Question 7

AB is a diameter of the circle *ABC*.



Not drawn accurately

Work out the value of k. *(5 marks)*

Question 8

A circle has equation $(x - 5)^2 + (y - 4)^2 = 100$

Show that the point (13, –2) lies on the circle. *(2 marks)*

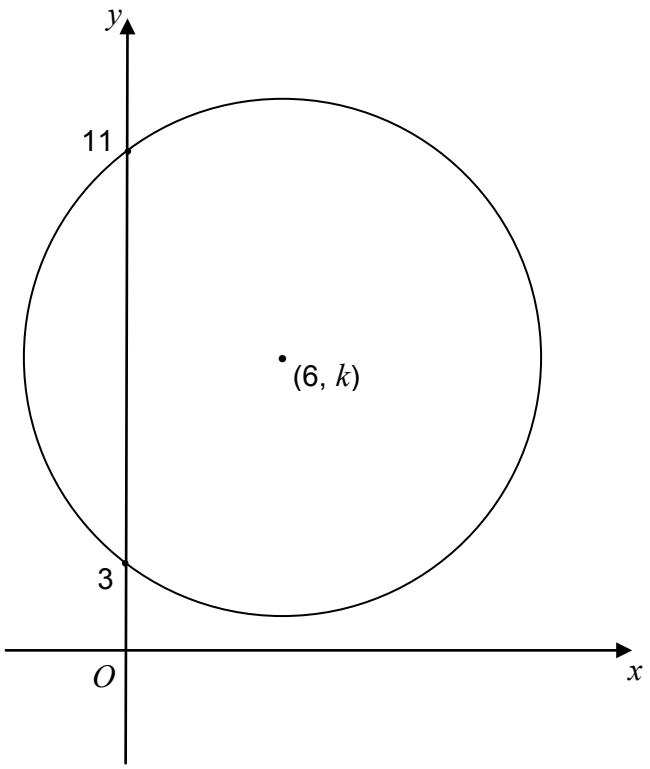
Question 9

The point (13, –2) lies on the circle $(x - a)^2 + (y - 4)^2 = 100$

Work out the two possible values of a. *(5 marks)*

Question 10

A circle passes through the points (0, 3) and (0, 11) and has centre (6, k)



Not drawn accurately

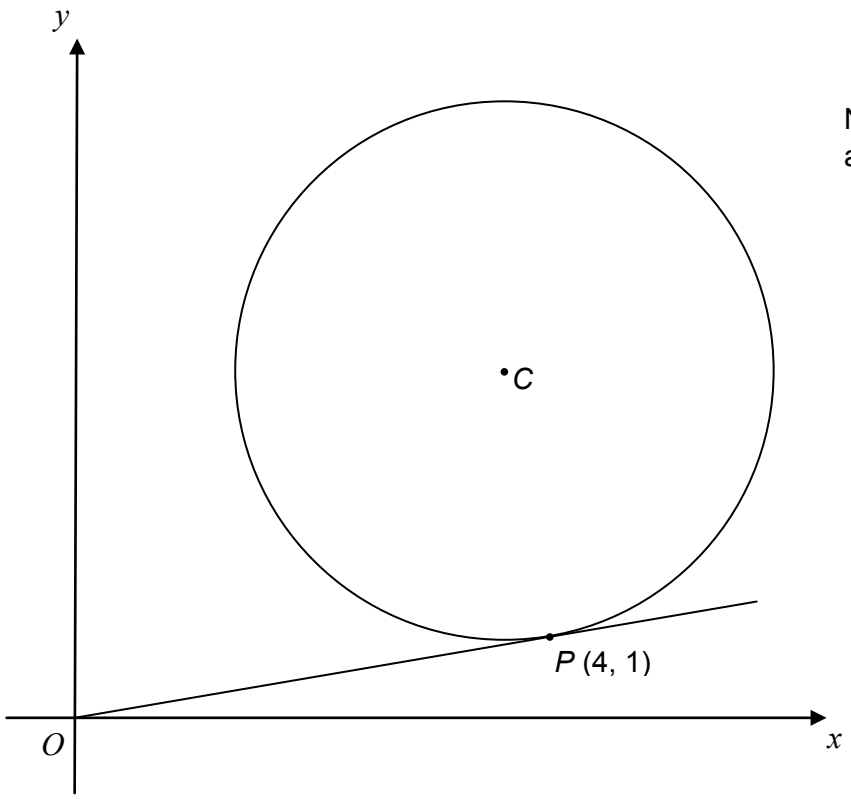
(a) Work out the value of k.

(b) Hence find the equation of the circle. *(5 marks)*

Question 11 (non-calculator)

The equation of this circle, centre *C*, is $(x - 3)^2 + (y - 5)^2 = 17$

P (4, 1) is a point on the circle.



Not drawn accurately

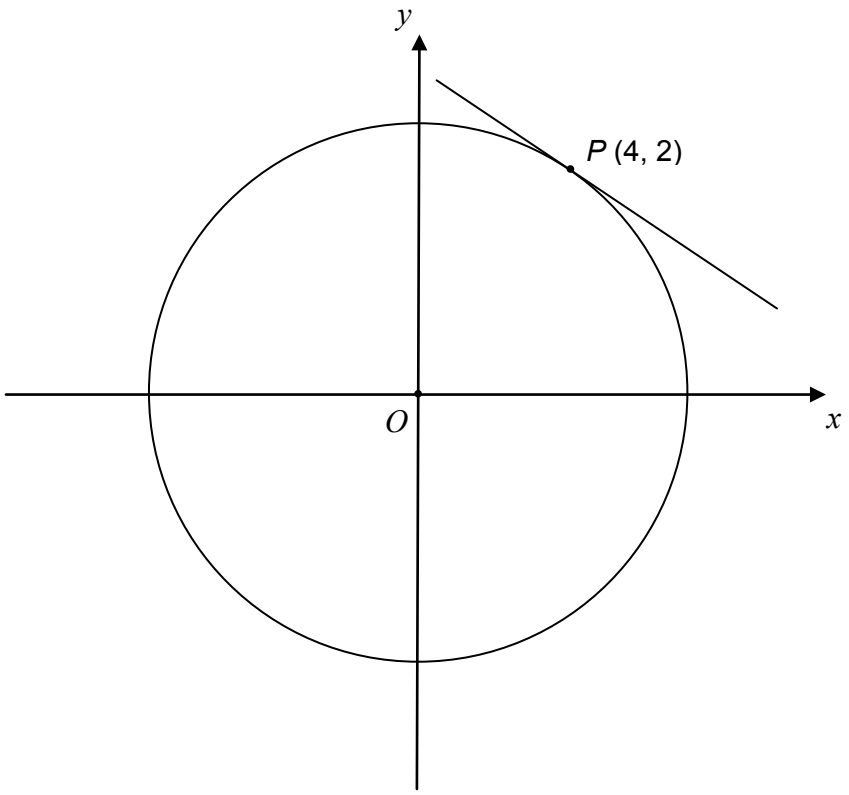
(a) Show working to explain why *OP* is a tangent to the circle. *(5 marks)*

(b) Show that the length *OP* is equal to the radius of the circle. (*3 marks*)

Question 12 (non-calculator)

The equation of this circle is $x^2 + y^2 = 20$

P (4, 2) is a point on the circle.



Not drawn accurately

Work out the equation of the tangent to the circle at *P*.

Give your answer in the form $y = mx + c$ (*3 marks*)

Question 13

A (–2, 5) and *B* (4, 13) are points on a circle.

AB is a diameter.

(a) Work out the equation of the circle.

(4 marks)

(b) Work out the equation of the tangent to the circle passing through *A*.

Give your answer in the form $ax + by + c = 0$

(4 marks)

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Level 2 (8360)

Worksheet 2

Geometric Problems and Proof

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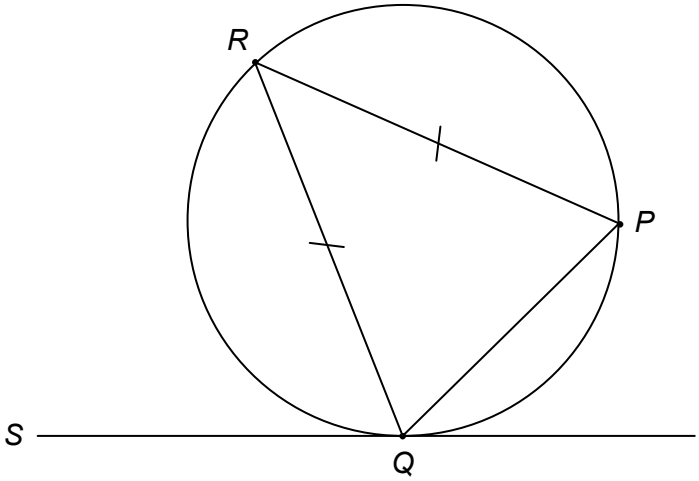
2 Geometric Problems and Proof

Question 1

SQ is a tangent to the circle at *Q*.

PR = *QR*



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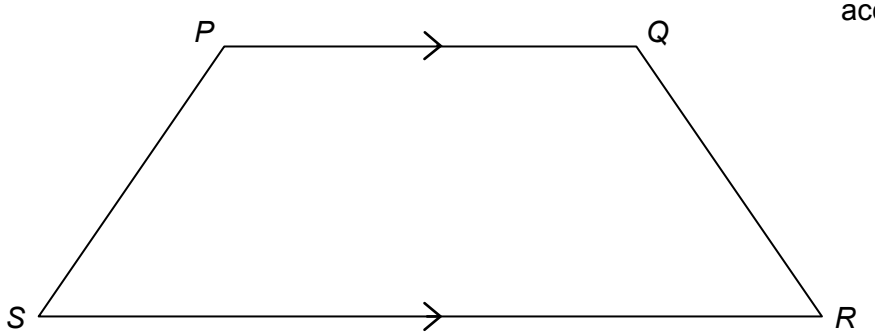
Prove that *RQ* bisects angle *PQS*. *(3 marks)*

Question 2

PQRS is a trapezium.

Angle *PSR* = angle *QRS*

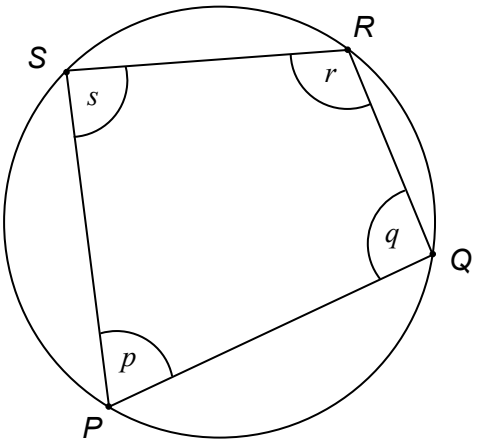


Not drawn accurately

Prove that *PQRS* is a cyclic quadrilateral. *(3 marks)*

Question 3

$p : q : r = 4 : 6 : 5$



Not drawn accurately

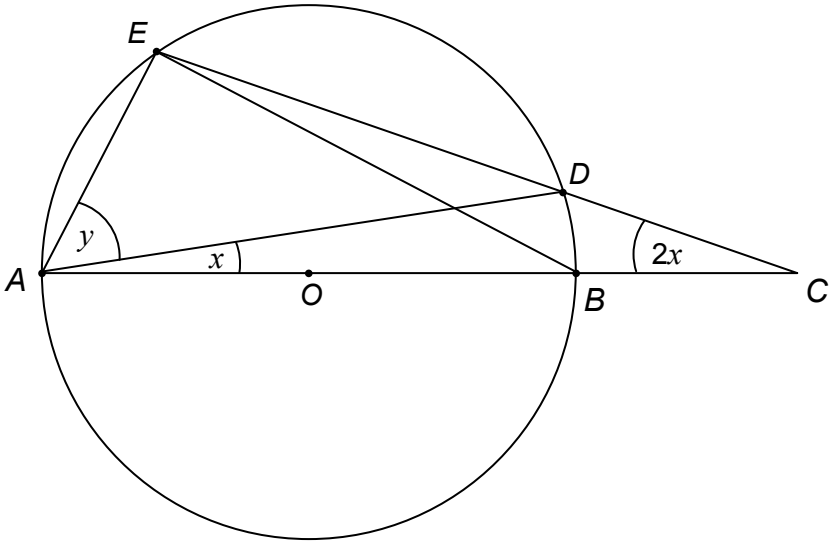
Work out s.

(5 marks)

Question 4

O is the centre of the circle.

$AOBC$ and EDC are straight lines.



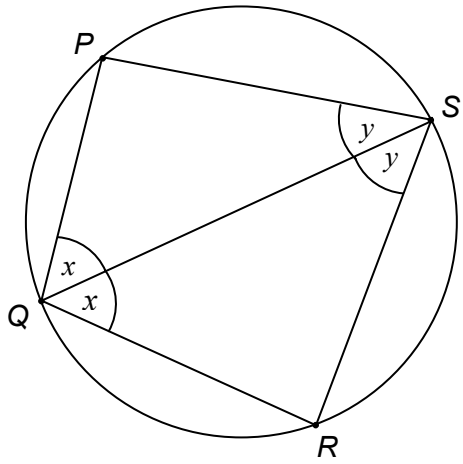
Not drawn accurately

Prove that $4x + y = 90$

(4 marks)

Question 5

QS bisects both of the angles *PSR* and *PQR*.



Not drawn accurately

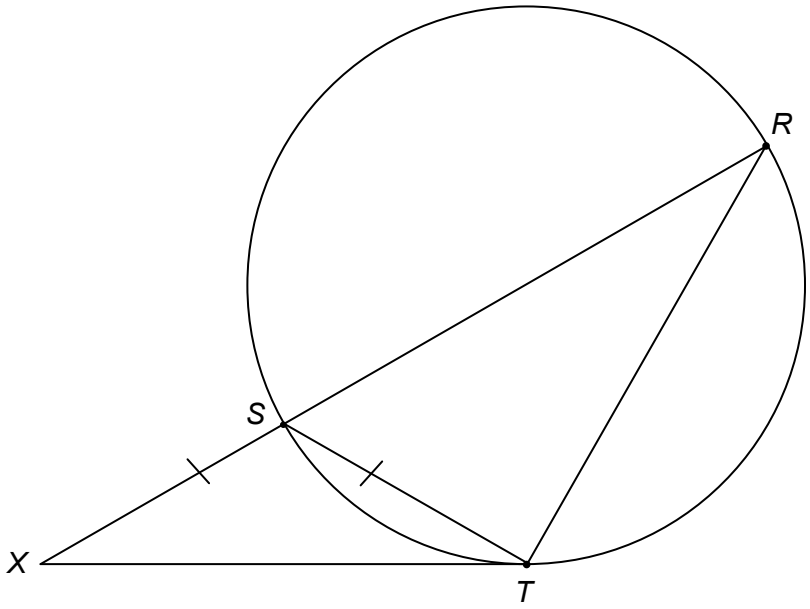
Prove that *QS* is a diameter of the circle. *(4 marks)*

Question 6

RSX is a straight line.

XT is a tangent to the circle at *T*.
SX = *ST*

Not drawn accurately



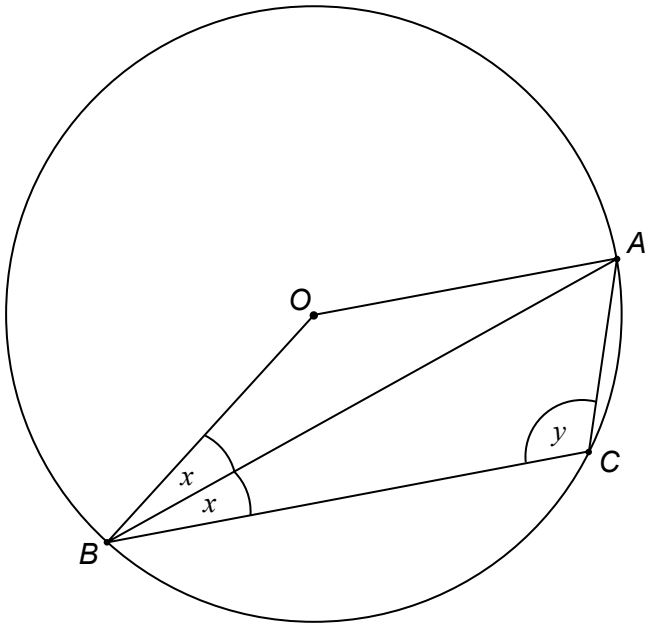
Prove that triangle *RXT* is isosceles. *(3 marks)*

Question 7

O is the centre of the circle.

AB bisects angle *OBC*.

Not drawn accurately



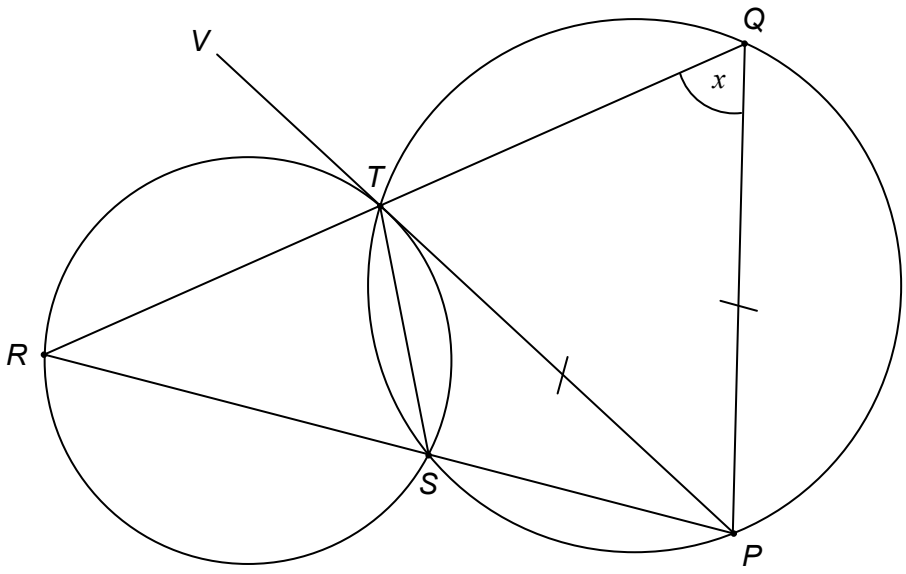
Prove that $y = 90 + x$ *(5 marks)*

Question 8

RTQ, RSP and *PTV* are all straight lines.

PT = *PQ*

Not drawn accurately



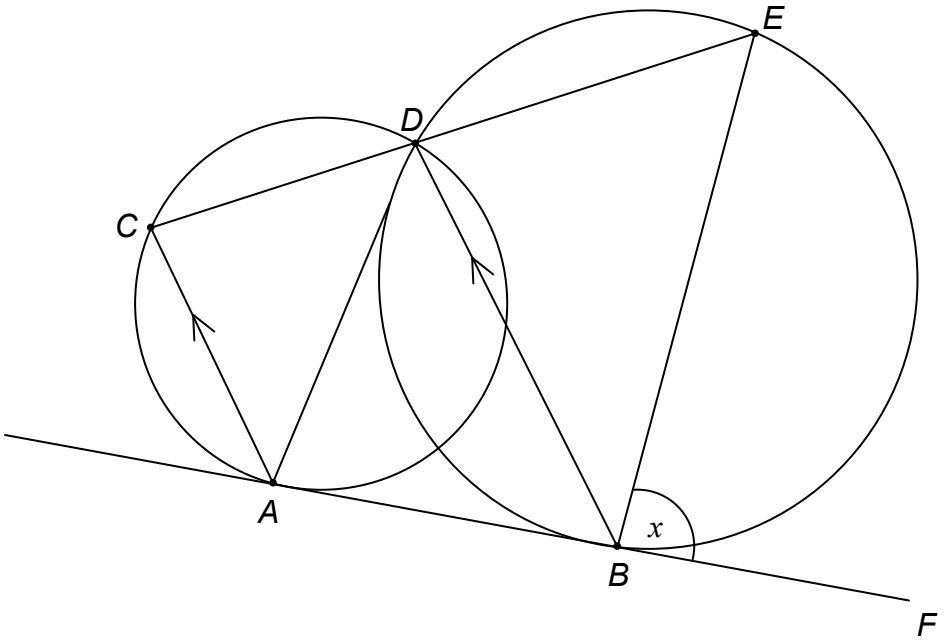
Prove that *PTV* is a tangent to circle *RST* at *T* *(5 marks)*

Question 9

ABF is a common tangent to the two circles at *A* and *B*.

CDE is a straight line.

AC is parallel to *BD*.



Not drawn accurately

Prove that *AD* is parallel to *BE*. *(5 marks)*

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Level 2 (8360)

Worksheet 3
Algebraic Proof

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3 Algebraic Proof

Question 1

Prove that $4(p-3)-2(2p-1)$ is always a negative integer. *(2 marks)*

Question 2

Prove that $8(y+3)+3(2-y)$ is a multiple of 5 when y is a positive integer. *(3 marks)*

Question 3

a is a positive integer.

Prove that $4a^2(2a+1)-(2a)^2$ is a cube number. *(3 marks)*

Question 4

a and b are positive integers.

$a < b$

Prove that $\frac{ax+3a}{bx+3b} < 1 \qquad x \neq -3$ *(3 marks)*

Question 5

(a) Express $x^2+6x+11$ in the form $(x+a)^2+b$ where a and b are integers. *(2 marks)*

(b) Hence, prove that $x^2+6x+11$ is always positive. *(2 marks)*

Question 6

Prove that, for all values of x, $x^2+2x+6 > 0$ *(4 marks)*

Question 7

$f(x) = (2x+3)^2+8(x+2)$ for all values of x.

Prove that there is exactly one value of x for which $f(x) = 0$ *(4 marks)*

Question 8

The nth term of a sequence is $\frac{1}{2}n(n+1)$

(a) Work out an expression for the $(n-1)$th term of the sequence.

Give your answer in its simplest form. *(2 marks)*

(b) Hence, or otherwise, prove that the sum of any consecutive pair of terms of the sequence is a square number. *(3 marks)*

Question 9

Prove that $\frac{x^2-4}{5x-10} \times \frac{10x^2}{x+2}$ is always positive. *(5 marks)*

Question 10

$f(n) = n^2 - n$

Prove that $f(3n) + f(n+1) = kn(5n-1)$ where k is an integer. *(3 marks)*

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Worksheet 4

Trigonometry

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4 Trigonometry

Question 1 (non-calculator)

Work out the exact value of $\sin 60° + \sin 120° + \sin 270°$.

Give your answer in its simplest form. *(3 marks)*

Question 2 (non-calculator)

Are these statements true or false?

	True	False
$\sin 37° = \sin 127°$	☐	☐
$\cos 54° = \cos 306°$	☐	☐
$\sin 135° = \cos 135°$	☐	☐
$\tan 126° = \tan 306°$	☐	☐

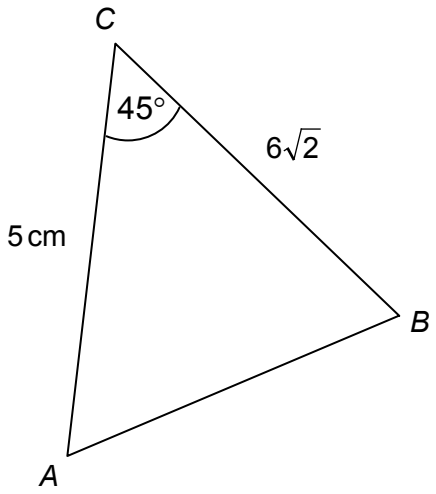
(4 marks)

Question 3 (non-calculator)

Work out the area of triangle *ABC*.

Write your answer in its simplest form.

Not drawn accurately



(3 marks)

Question 4 (calculator or non-calculator)

Show that $\tan^2\theta \equiv \frac{1}{\cos^2\theta} - 1$

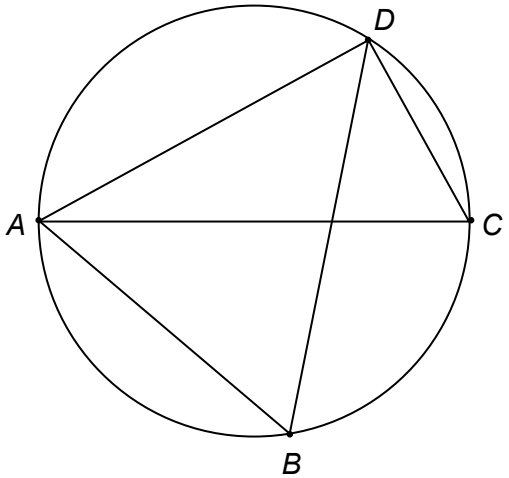
(3 marks)

Question 5 (calculator)

AC is a diameter of the circle.

AC = 5 cm, *AD* = 4 cm

Not drawn accurately



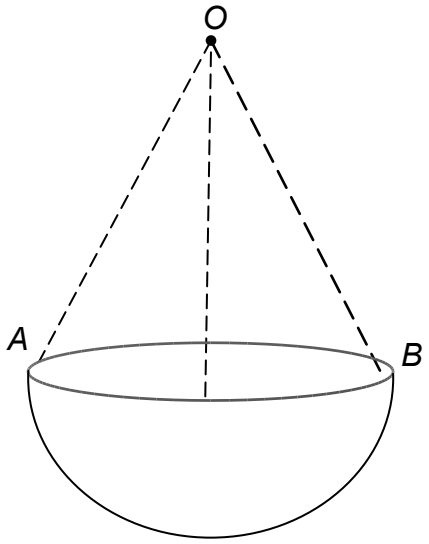
Work out angle *ABD*.

(4 marks)

Question 6 (calculator)

A hanging basket is made from a hemisphere and three chains.
The radius of the hemisphere is 10 cm.
Each chain is 30 cm long.
The chains are equally spaced around the rim of the hemisphere.

Work out angle *AOB*.



(5 marks)

Question 7 (calculator)

Solve the following equation for $0 < \theta < 360°$.

$\tan^2 \theta = 2$

(4 marks)

Question 8 (calculator)

Solve the following equation for $0 < \theta < 360°$.

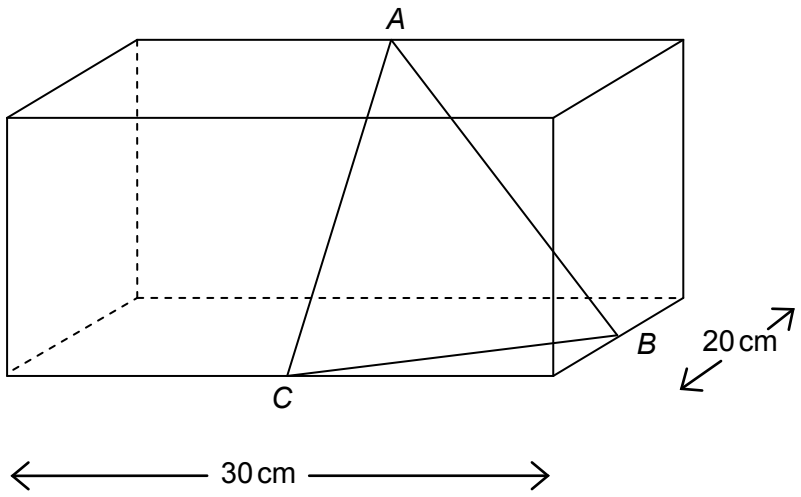
$3\cos^2 \theta + 2\cos \theta - 1 = 0$

(5 marks)

Question 9 (calculator)

A cuboid has length 30 cm and width 20 cm

A, *B* and *C* are midpoints of three of the edges.



(a) Work out the distance BC.

(2 marks)

(b) Given that angle *ACB* = 59° and *AB* = 22 cm

work out the size of angle *CAB*.

(3 marks)

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Worksheet 5

Matrices 1

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5 Matrices 1

Question 1

Work out

(a) $\begin{pmatrix} 4 & 2 \\ -3 & 5 \end{pmatrix}\begin{pmatrix} 7 \\ 1 \end{pmatrix}$

(b) $\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}\begin{pmatrix} -3 \\ -4 \end{pmatrix}$

(c) $2\begin{pmatrix} 5 & -2 \\ 6 & -3 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

(e) $6\begin{pmatrix} -4 & 7 \\ -1 & -3 \end{pmatrix}$

(f) $\begin{pmatrix} 8 & 4 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} -3 \\ 6 \end{pmatrix}$

(12 marks)

Question 2

Work out

(a) $\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 0 & 3 \\ 1 & -4 \end{pmatrix}$

(b) $\begin{pmatrix} -3 & -2 \\ -1 & 5 \end{pmatrix}\begin{pmatrix} -2 & 4 \\ 3 & 4 \end{pmatrix}$

(c) $\begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}\begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix}$

(d) $\begin{pmatrix} 10 & -7 \\ 9 & 8 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ -2 & 3 \end{pmatrix}$

(e) $\begin{pmatrix} 1 & -2 \\ 3 & -5 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$

(f) $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ 3 & -5 \end{pmatrix}$

(12 marks)

Question 3 (non-calculator)

Work out, giving your answers as simply as possible.

(a) $\begin{pmatrix} \sqrt{2} & 1 \\ -1 & 3\sqrt{2} \end{pmatrix}\begin{pmatrix} \sqrt{2} & 0 \\ -3 & -2\sqrt{2} \end{pmatrix}$

(b) $\begin{pmatrix} -\frac{1}{2} & -1 \\ \frac{3}{2} & 5 \end{pmatrix}\begin{pmatrix} -2 & 4 \\ -\frac{1}{2} & 3 \end{pmatrix}$

(c) $\begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}^2$

(d) $\begin{pmatrix} 3\sqrt{3} & -4 \\ 2 & 3\sqrt{3} \end{pmatrix}\begin{pmatrix} \sqrt{3} & 1 \\ -4 & 0 \end{pmatrix}$

(e) $\begin{pmatrix} \frac{1}{3} & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{4} \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$

(f) $\begin{pmatrix} \sqrt{2} & 2 \\ 7 & \sqrt{3} \end{pmatrix}^2$

(17 marks)

Question 4

Work out, giving your answers as simply as possible.

(a) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} p \\ p+1 \end{pmatrix}$

(b) $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} m \\ 2m \end{pmatrix}$

(d) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} -a & 0 \\ 0 & a \end{pmatrix}$

(e) $\begin{pmatrix} 4t & 0 \\ 0 & 4t \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

(f) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

(13 marks)

Question 5

Work out, giving your answers as simply as possible.

(a) $\begin{pmatrix} 2x & -3 \\ -5 & 4x \end{pmatrix}\begin{pmatrix} x & 3x \\ -3 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} a & 3a \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 7 & 8 \\ -10 & 11 \end{pmatrix}$

(c) $\begin{pmatrix} x & 0 \\ 1 & x \end{pmatrix}^2$

(d) $\begin{pmatrix} y & y \\ -3 & x \end{pmatrix}\begin{pmatrix} 2 & 3y \\ 0 & 1 \end{pmatrix}$

(e) $\begin{pmatrix} a+1 & a \\ a+2 & a+1 \end{pmatrix}\begin{pmatrix} a+1 & -a \\ -a-2 & a+1 \end{pmatrix}$

(f) $\begin{pmatrix} 3x & -3 \\ -9 & x+1 \end{pmatrix}^2$

(14 marks)

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Worksheet 6

Matrices 2

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6 Matrices 2

Question 1

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 7 & 4 \\ 5 & 3 \end{pmatrix} \qquad \mathbf{C} = \begin{pmatrix} -2 & 3 \\ 1 & -1 \end{pmatrix}$$

Work out

(a) **AB**

(b) **BC**

(c) 3**A**

(d) **BA**

(e) –**C**

(f) $\mathbf{B}\begin{pmatrix} 1 & -4 \\ -5 & 7 \end{pmatrix}$

(12 marks)

Question 2

$$\mathbf{P} = \begin{pmatrix} -2 & 0 \\ 5 & 1 \end{pmatrix} \qquad \mathbf{Q} = \begin{pmatrix} -4 & 1 \\ 3 & -2 \end{pmatrix} \qquad \mathbf{C} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

Work out

(a) $\mathbf{P}^2$

(b) **QP**

(c) 5**Q**

(d) **PC**

(e) **IQ**

(f) 3**I**

(12 marks)

Question 3

$$\begin{pmatrix} -2 & a \\ -4 & 3 \end{pmatrix}\begin{pmatrix} 3 \\ 7 \end{pmatrix} = \begin{pmatrix} 22 \\ 9 \end{pmatrix}$$

Work out the value of a.

(2 marks)

Question 4

Work out the values of a, b and c.

$$\begin{pmatrix} 2 & a \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 2 & b \end{pmatrix} = \begin{pmatrix} 12 & 26 \\ c & 13 \end{pmatrix}$$

(3 marks)

Question 5

Work out the image of the point D (–1, 2) after transformation by the matrix $\begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$

(2 marks)

Question 6

The point $A(m, n)$ is transformed to the point A' (–2, 0) by the matrix $\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$

Work out the values of m and n.

(4 marks)

Question 7

The matrix A represents a reflection in the line $y = x$.

Write down the matrix A.

The unit square is transformed by the matrix A and then by rotation through –90° about O.

Work out the matrix representing the combined transformation.

(4 marks)

Question 8

Describe fully the transformation given by the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

(2 marks)

Question 9 (non-calculator)

The unit square $OABC$ is transformed by the matrix $\begin{pmatrix} h & 0 \\ 0 & h \end{pmatrix}$ to the square $OA'B'C'$.

The area of $OA'B'C'$ is 27.

Work out the exact value of h.

(3 marks)

Question 10

$\mathbf{A} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

The point P (2, 7) is transformed by matrix **BA** to **P′**.

Show that P lies on the line $7x + 2y = 0$

(3 marks)

AQA Qualifications

AQA Level 2 Certificate FURTHER MATHEMATICS

Level 2 (8360)

Worksheet 7

Inequalities

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7 Inequalities

Question 1

$-6 < 3x \leq 6$

x is an integer

Write down all the possible values for x. *(2 marks)*

Question 2

Solve $6x > 24 - 2x$ *(2 marks)*

Question 3

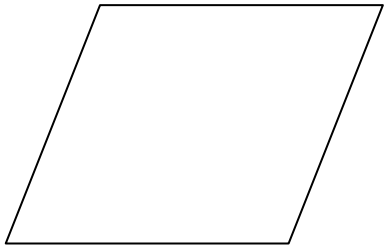
Solve $4(2x - 1) < 2$ *(3 marks)*

Question 4

A rhombus and a rectangle are shown.

The perimeter of the rhombus is greater than the perimeter of the rectangle.

Not drawn accurately



$2y + 6$

$y + 4$

$2y + 10$

Show that $y > k$ where k is an integer. *(4 marks)*

Question 5

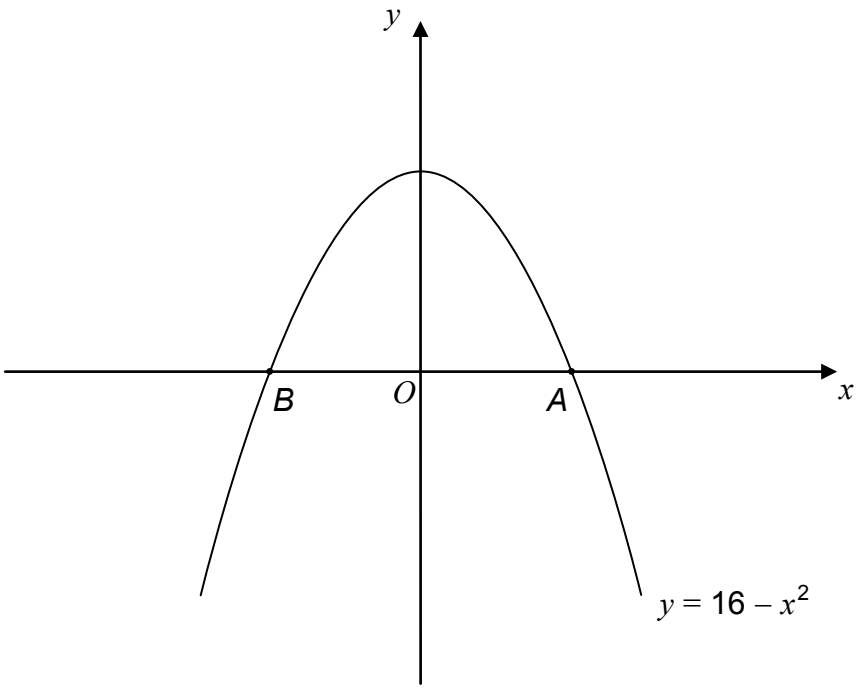
$p < -1$ and $q > 1$

Tick the correct box for each statement.

	Always true	Sometimes true	Never true
$5p < 0$	☐	☐	☐
$p^2 < 0$	☐	☐	☐
$p + q > 0$	☐	☐	☐
$-1 < \frac{q}{p} < 0$	☐	☐	☐

(4 marks)

Question 6



(a) Write down the coordinates of points *A* and *B*. *(2 marks)*

(b) Hence, or otherwise, solve $16 - x^2 \geq 0$ *(2 marks)*

Question 7

(a) Factorise $x^2 + 3x$ *(1 mark)*

(b) Sketch $y = x^2 + 3x$

Label the x values of the points of intersection with the x-axis. *(2 marks)*

(c) Hence, or otherwise, solve $x^2 + 3x < 0$ *(2 marks)*

Question 8

Solve $(x - 5)(x + 2) \geq 0$ *(3 marks)*

Question 9

Solve $x^2 + 4x - 12 < 0$ *(4 marks)*

Question 10

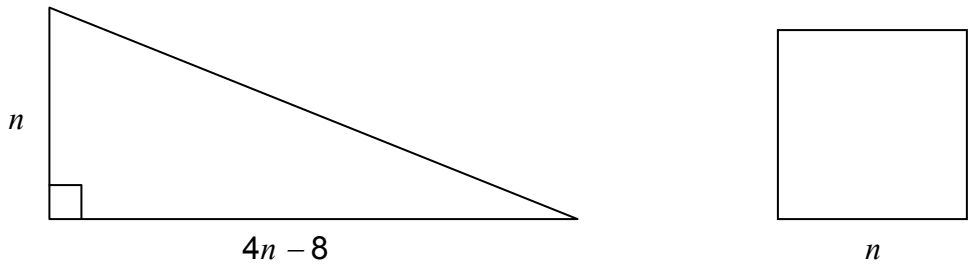
Solve $2x^2 - x - 3 < 0$ *(4 marks)*

Question 11

Solve $3x^2 > 14x - 8$ *(4 marks)*

Question12

A triangle and a square are shown.



Work out the range of values of n for which

area of triangle < area of square

(5 marks)

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Level 2 (8365)

Worksheet 8
Functions

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8 Functions

Question 1 (non-calculator)

$f(x) = 2x^3 - 250$

Work out x when $f(x) = 0$ *(3 marks)*

Question 2

$f(x) = x^2 + ax - 8$

$f(-3) = 13$

Work out the value of a. *(3 marks)*

Question 3

$f(x) = x^2 + 3x - 10$

Show that $f(x + 2) = x(x + 7)$ *(3 marks)*

Question 4

Work out the range for each of these functions.

(a) $f(x) = x^2 + 6$ for all x *(1 mark)*

(b) $f(x) = 3x - 5$ $-2 \leqslant x \leqslant 6$ *(2 marks)*

(c) $f(x) = 3x^4$ $x < -2$ *(1 mark)*

Question 5

(a) The function $= \frac{x + 2}{x - 3}$

Give a reason why $x > 0$ is **not** a suitable domain for $f(x)$. *(1 mark)*

(b) Give a possible domain for $f(x) = \sqrt{x - 5}$ *(1 mark)*

Question 6

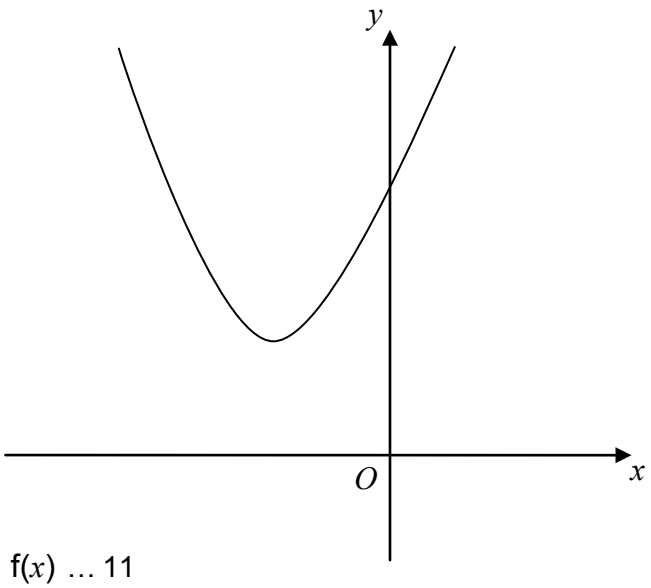
$f(x) = 3 - 2x$ $\quad a < x < b$

The range of $f(x)$ is $-5 < f(x) < 5$

Work out a and b. *(3 marks)*

Question 7

Here is a sketch of $f(x) = x^2 + 6x + a$ for all x, where a is a constant



The range of $f(x)$ is $f(x) \ldots 11$

Work out the value of a. *(3 marks)*

Question 8

(a) Factorise $x^2 - 5x - 14$ *(2 marks)*

(b) Sketch the function $f(x) = x^2 - 5x - 14$ for all x.

Label the points of intersection with the x and y axes. *(3 marks)*

Question 9

$f(x) = -x^2$ $\quad 0 \leqslant x < 2$

-4 $\quad 2 \leqslant x < 3$

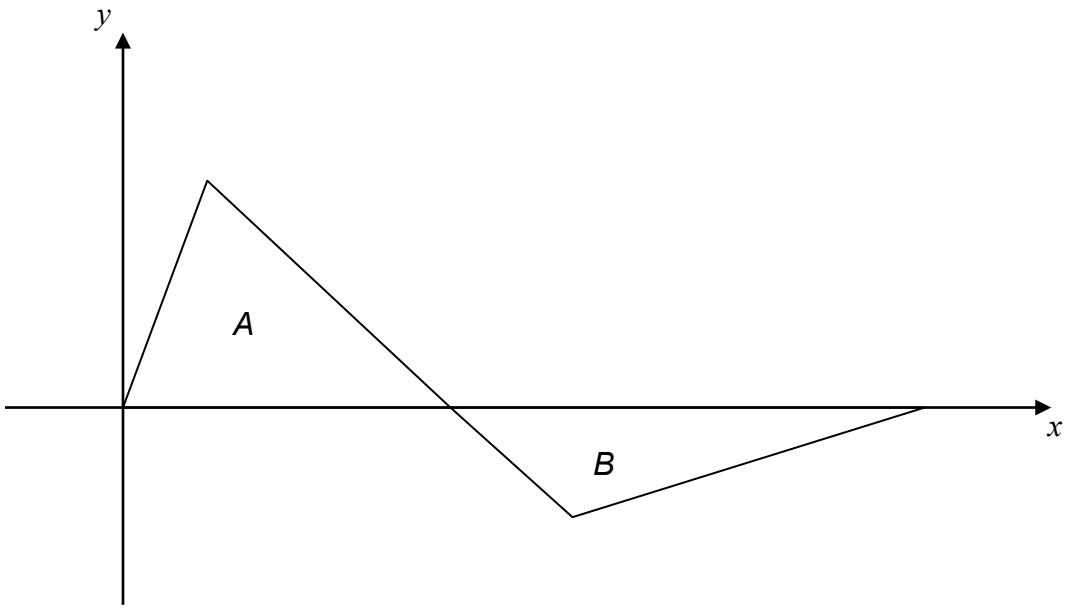
$2x - 10$ $\quad 3 \leqslant x \leqslant 5$

Draw the graph of $f(x)$ for values of x from 0 to 5 *(3 marks)*

Question 10

Here is a sketch of y = f(x) for values of x from 0 to 7.

$$f(x) = \begin{cases} 2x & 0 \leqslant x < 1 \\ 3 - x & 1 \leqslant x < 4 \\ \frac{x-7}{3} & 4 \leqslant x \leqslant 7 \end{cases}$$



Show that

area of triangle *A* : area of triangle *B* = 3 : 2 *(4 marks)*

Question 11

$f(x) = \dfrac{\sqrt{x} - a}{2}$ for $x > 0$, where a is a positive constant.

If $f^{-1}(3a) = 306.25$ work out the value of a

(4 marks)

Question 12

$f(x) = \dfrac{2x-1}{4}$ $g(x) = \dfrac{5}{x+1}$

Work out $fg(x)$

Give your answer in the form $\dfrac{ax+b}{cx+d}$ where a, b, c and d are integers.

(2 marks)

Question 13

y = f(x) is the graph of a function.

$\frac{dy}{dx} = (x - 5)(2x + 1)$

Work out the vales of x for which the function is decreasing.

Give your answer as an inequality.

(2 marks)

AQA Qualifications

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Level 2 (8360)

Worksheet 9

Coordinate Geometry - Calculus

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9 Coordinate Geometry – Calculus

Question 1

For each of these straight lines, work out

(i) The gradient of the line *(1 mark for each part)*

(ii) The gradient of the line that is perpendicular to the given line *(1 mark for each part)*

(iii) The y-intercept of the line *(1 mark for each part)*

(a) $y = 5x - 4$ (b) $3y = 9 - 6x$ (c) $3y - 12 = 2x$

(d) $5x - 2y + 15 = 0$ (e) $\frac{x}{4} - \frac{y}{3} = 2$

Question 2

For each of these straight line segments, AB, work out

(i) The mid-point of AB *(2 marks for each part)*

(ii) The gradient of AB *(1 mark for each part)*

(iii) The length of AB, giving your answer as an integer or a surd *(2 marks for each part)*

(a) $A = (-3, -4)$ $B = (4, 3)$ (b) $A = (-4, 1)$ $B = (1, 5)$ (c) $A = (5, -2)$ $B = (0, 10)$

(d) $A = (-2, -6)$ $B = (-6, 0)$ (e) $A = (1, 9)$ $B = (9, -6)$ (f) $A = (7, 1)$ $B = (-5, -3)$

Question 3

In each of these line segments, B lies between A and C.

Work out the coordinates of C in each case. *(2 marks for each part)*

(a) $A = (-1, 3)$ $B = (1, 1)$ and $AB : BC = 1 : 2$

(b) $A = (-4, -2)$ $B = (2, -5)$ and $AB : BC = 3 : 1$

(c) $A = (11, 0)$ $B = (1, -5)$ and $AB : BC = 5 : 3$

(d) $A = (-6, 2)$ $B = (0, 4)$ and $AB : BC = 2 : 3$

(e) $A = (2, -9)$ $B = (-3, 1)$ and $AB : BC = 5 : 4$

Question 4

Work out the coordinates of the points of intersection of the curve $y = x^2 + 7$ and the straight line $y = 5x + 1$ *(4 marks)*

Question 5

Line L has equation $y + 3x = 7$

Line N is perpendicular to line L and passes through $(3, -1)$.

Work out the equation of line N.

Give your answer in the form $y = ax + b$ *(4 marks)*

Question 6

Work out $\frac{dy}{dx}$ for each of the following

(a) $y = 7x + 3$ *(1 mark)*

(b) $y = 8 - 5x + x^2$ *(2 marks)*

(c) $y = 3x^3 + 4x$ *(2 marks)*

(d) $y = x^3 - 7x^2 + 10x - 1$ *(2 marks)*

(e) $y = 4x(x^2 + 2x - 3)$ *(3 marks)*

(f) $y = (3x - 5)(x + 8)$ *(3 marks)*

(g) $y = x(7 - x)(6 - 2x)$ *(3 marks)*

(h) $y = (x + 3)(x - 1)(x - 6)$ *(4 marks)*

Question 7

A curve has equation $y = x^3 + x^2 + 2x - 4$

Work out the equation of the tangent to this curve where $x = -2$

Give your answer in the form $y = ax + b$ *(5 marks)*

Question 8

A curve has equation $y = x^3 + 2x^2 - 9x + 3$

Work out the equation of the normal to this curve at the point (1, –3)

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. *(5 marks)*

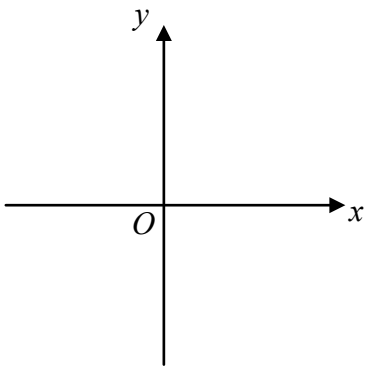
Question 9

A curve has equation $y = x^3 - 6x^2 + 20$

(a) Write down an expression for $\frac{dy}{dx}$ *(1 mark)*

(b) Work out the coordinates of the points at which the gradient is zero and determine whether they are maximum or minimum. *(5 marks)*

(c) Sketch the curve on the axes clearly labelling the maximum and minimum points. *(2 marks)*



Question 10

A curve has equation $y = x^3 - x^2 + kx - 2$

(a) Write down an expression for $\frac{dy}{dx}$ *(1 mark)*

(b) The curve has a minimum point at the point where $x = 2$

Work out the value of k. *(2 marks)*

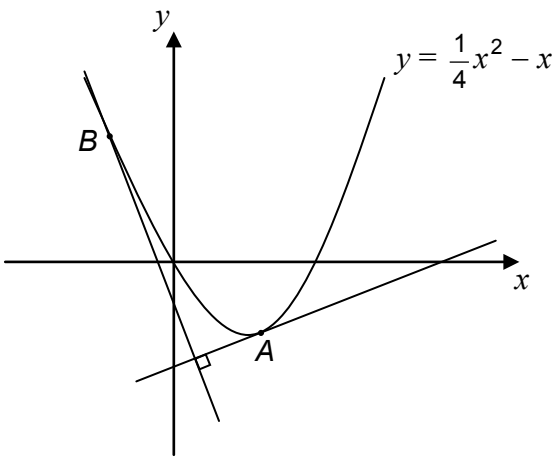
(c) Work out the x coordinate of the maximum point on the curve. *(3 marks)*

Question 11

(a) Show that the line $y = \frac{1}{2}x - \frac{9}{4}$ is the tangent to the curve $y = \frac{1}{4}x^2 - x$

at the point A $(3, -\frac{3}{4})$. *(4 marks)*

(b) The point *B* on the curve is such that the tangent at *B* is perpendicular to the tangent at *A*, as shown in the diagram.



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Work out the coordinates of *B*. *(4 marks)*

Question 12

Work out $\frac{dy}{dx}$ for each of the following

(a) $y = 3x^{-2} + 3$ *(1 mark)*

(b) $y = 5x^{-1} + 2x^2$ *(2 marks)*

(c) $y = 3x^{-3} - 4x^{-5}$ *(2 marks)*

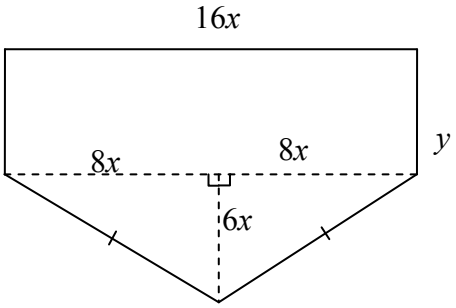
(d) $y = \frac{5 + x}{x^2}$ *(2 marks)*

(e) $y = \frac{1}{x}(x^4 + 2x - 4)$ *(3 marks)*

(f) $y = \frac{3x + 2x^6}{4x^3}$ *(3 marks)*

Question 13

A pentagon is made from a rectangle and an isosceles triangle.



(a) The perimeter of the pentagon is 84 cm

Show that $y = 42 - 18x$

(*2 marks*)

(b) Show that the area, A cm^2, of the pentagon is given by

$A = 672x - 240x^2$

(*2 marks*)

(c) Using calculus, work out the maximum value of A as x varies.

(*3 marks*)

Question 13

The curve $y = \frac{x}{4} + \frac{8}{x^2}$ has a minimum point

Work out this minimum value of y.

(4 marks)

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Level 2 (8365)

Worksheet 10
Factor Theorem

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10 Factor Theorem

Question 1

(a) Show that $x(x + 4)(x - 9) = x^3 - 5x^2 - 36x$ *(1 mark)*

(b) Write down the x values of the three points where the graph of $y = x^3 - 5x^2 - 36x$ crosses the x-axis. *(2 marks)*

Question 2

$f(x) = x^3 + 2x^2 - 5x - 6$

(a) Work out f(1) and f(–1) *(2 marks)*

(b) Work out f(2) and f(–2) *(2 marks)*

(c) Work out f(3) and f(–3) *(2 marks)*

(d) Write down the three linear factors of f(x). *(1 mark)*

Question 3

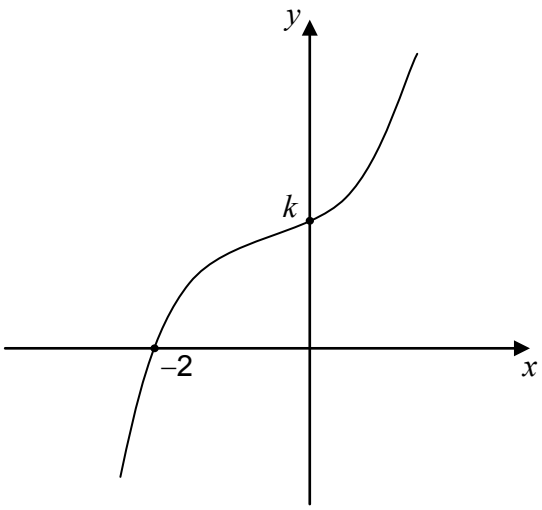
(a) Show that $(x + 5)$ is a factor of $x^3 + 7x^2 + 2x - 40$ *(2 marks)*

(b) Work out the other two linear factors of $x^3 + 7x^2 + 2x - 40$ *(3 marks)*

(c) Hence, solve $x^3 + 7x^2 + 2x - 40 = 0$ *(1 mark)*

Question 4

A sketch of $y = x^3 + 5x^2 + 9x + k$ where k is an integer, is shown.



Work out the value of k. *(3 marks)*

Question 5

(a) $(x + 3)$ is a factor of $f(x) = x^3 + x^2 + ax - 72$ where a is an integer.

Work out the value of a. *(3 marks)*

(b) Work out the other linear factors of $f(x)$. *(3 marks)*

Question 6

$(x - 3)$ and $(x + 4)$ are factors of $f(x) = x^3 + ax^2 + bx + 24$ where *a* and *b* are integers.

(a) Work out the third linear factor of $f(x)$. *(2 marks)*

(b) Work out the values of a and b. *(4 marks)*

Question 7

(a) $(x - 5)$ is a factor of $f(x) = x^3 + kx^2 + 9x - 20$ where k is an integer.

Work out the value of k. *(3 marks)*

(b) Express $f(x)$ as a product of $(x - 5)$ and a quadratic factor. *(2 marks)*

(c) Show that $(x - 5)$ is the only linear factor of $f(x)$. *(2 marks)*

Question 8

Solve $x^3 - 6x^2 - 25x - 18 = 0$ *(5 marks)*

Question 9

f(x) = $x^5 - 2x^4 - 81x + 162 = 0$

(a) Use the factor theorem to show that f(x) has a factor of $(x - 2)$

(1 mark)

(b) Hence work out the integer solutions of f(x) = 0

(4 marks)

Question 10

(a) Use the factor theorem to show that $(3x + 2)$ is a factor of $3x^3 + 2x^2 - 3x - 2$

(2 marks)

(b) Factorise fully $3x^3 + 2x^2 - 3x - 2$

(2 marks)

AQA Qualifications

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Level 2 (8360)

Worksheet 11

Sequences

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11 Sequences

Question 1

A linear sequence starts

250 246 242 238 .....

Which term is the first to have a negative value? *(4 marks)*

Question 2

Work out the nth term of this quadratic sequence.

8 9 14 23 36 .....

(4 marks)

Question 3

(a) Show that the nth term of the quadratic sequence

4 10 18 28 ..... is $n^2 + 3n$

(3 marks)

(b) Hence, write down the nth term of these quadratic sequences.

(b) (i) 5 11 19 29 .....

(1 mark)

(b) (ii) 5 12 21 32 .....

(1 mark)

Question 4 (non calculator)

(a) Write down the nth term of the linear sequence

4 7 10 13 ……

(1 mark)

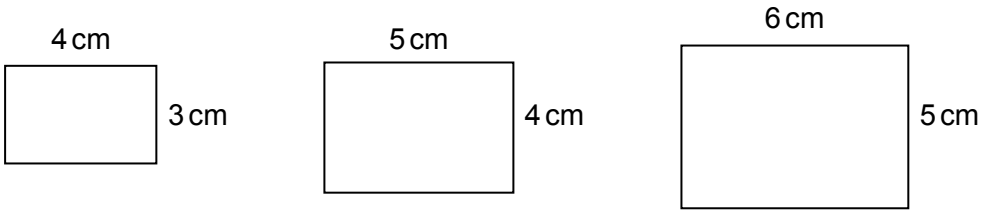
(b) Hence, write down the nth term of the quadratic sequence.

16 49 100 169 ……

(1 mark)

(c) For the sequence in part 4(b), show that the 30th term is equal to the product of the 2nd and 4th terms *(3 marks)*

Question 5



This pattern of rectangles continues.

Show that the sequence of numbers formed by the areas of these rectangles has nth term

$n^2 + 5n + 6$ *(4 marks)*

Question 6

A linear sequence starts

$a + b$ $a + 3b$ $a + 5b$ $a + 7b$ ……

The 5th and 8th terms have values 35 and 59.

(a) Work out a and b. *(4 marks)*

(b) Work out the nth term of the sequence. *(2 marks)*

Question 7

A sequence has nth term $\dfrac{3n+1}{n}$

(a) Show that the difference between the nth and $(n+1)$th terms is $\dfrac{1}{n(n+1)}$ *(3 marks)*

(b) Which are the first two consecutive terms with a difference less than 0.01? *(2 marks)*

(c) Write down the limiting value of the sequence as $n \to \infty$ *(1 mark)*

Question 8

A sequence has nth term $\dfrac{5n+2}{2n}$

Show that the limiting value of the sequence, S, as $n \to \infty$ is 2.5 *(2 marks)*

Question 9

Here is the sequence of odd numbers

1 3 5 7 9 ……

A quadratic sequence is formed by multiplying consecutive odd numbers in successive pairs.

3 15 35 63 ……

Work out the nth term of this sequence. *(3 marks)*

Question 10

The nth term of a sequence is $\dfrac{2n^2-1}{3n^2+2}$

(a) Show that the difference between the first two terms is $\dfrac{3}{10}$ *(3 marks)*

(b) Write down the limiting value of the sequence as $n \to \infty$ *(1 mark)*

AQA Qualifications

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Worksheet 12
Algebraic Problems – including ratio

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12 Algebraic Problems – including ratio

Note

- If $x : y = 4 : 7$, then $\frac{x}{y} = \frac{4}{7}$
- If, in a problem, two numbers are in the ratio 4 : 7, use $4x$ and $7x$ as the numbers (usually leading to a linear equation); otherwise, use x and y as the numbers (which will lead to simultaneous equations).
- If $x : y = 4 : 7$, what is $x + 2y : 3x$?

  Think in terms of 'parts', ie 4 parts and 7 parts, so $x + 2y : 3x = 4 + 14 : 12$

  $= 18 : 12$

  $= 3 : 2$

Question 1

Work out the possible values of $\frac{2n - 1}{3n + 2}$ if $n^2 = 16$

Give your answers as fractions in their simplest form. *(4 marks)*

Question 2

$x : y = 6 : 5$

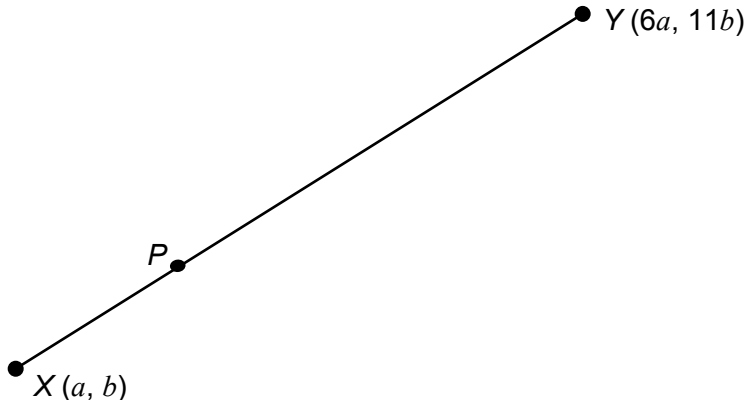
(a) Express x in terms of y. *(2 marks)*

(b) Show that $x + 3y : 2x - y = 3 : 1$ *(2 marks)*

Question 3

A point P divides XY in the ratio 3 : 7

Not drawn accurately



Work out the coordinates of P, in terms of a and b. *(3 marks)*

Question 4

Here is a linear sequence

$$a + b \quad a + 3b \quad a + 5b \quad a + 7b \quad \ldots\ldots$$

Given that

- 2nd term : 4th term = 2 : 5
- 1st term = – 4

Work out a and b. *(5 marks)*

Question 5

You are given that $ab + a = 5$ and $a : b = 4 : 3$

Work out the possible pairs of values of a and b. *(7 marks)*

Question 6

The sum of the ages of two people is 90 years.

Six years ago, their ages were in the ratio 8 : 5

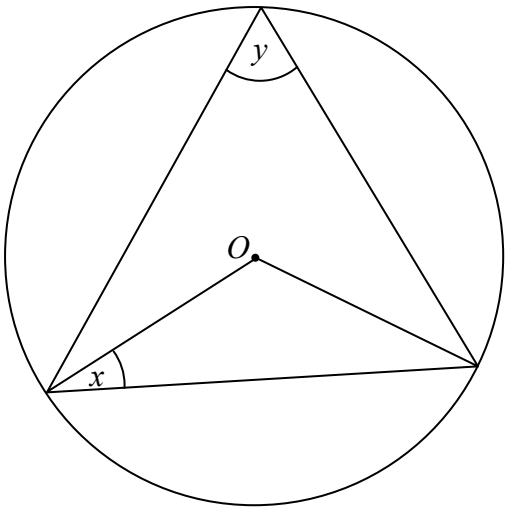
How old are they now?

Do **not** use trial and improvement.

You **must** show your working. *(5 marks)*

Question 7

O is the centre of the circle.



Not drawn accurately

Given that $x : y = 4 : 5$

Work out the value of y.

Do **not** use trial and improvement.

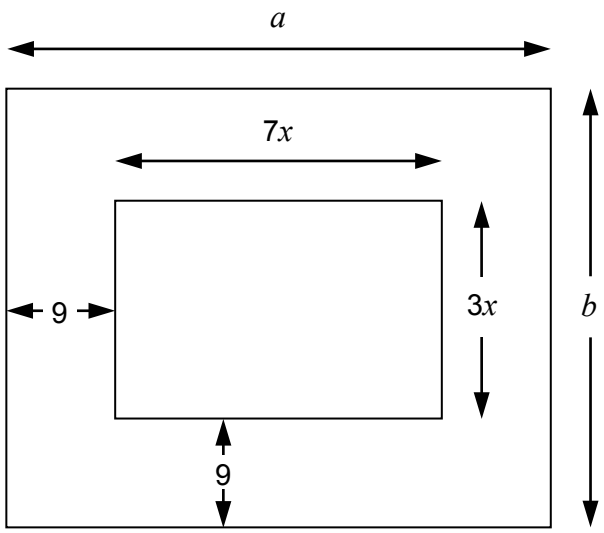
You **must** show your working.

(7 marks)

Question 8

A rectangular picture is surrounded by a frame of constant width.

All measurements are in centimetres.



Not drawn accurately

Given that $a : b = 3 : 2$

Work out x.

(5 marks)

Question 9

If $x : y = 3 : 5$ and $y : z = 10 : 9$

Find, in its simplest form

(a) $x : z$ *(3 marks)*

(b) $10x : 7y$ *(2 marks)*

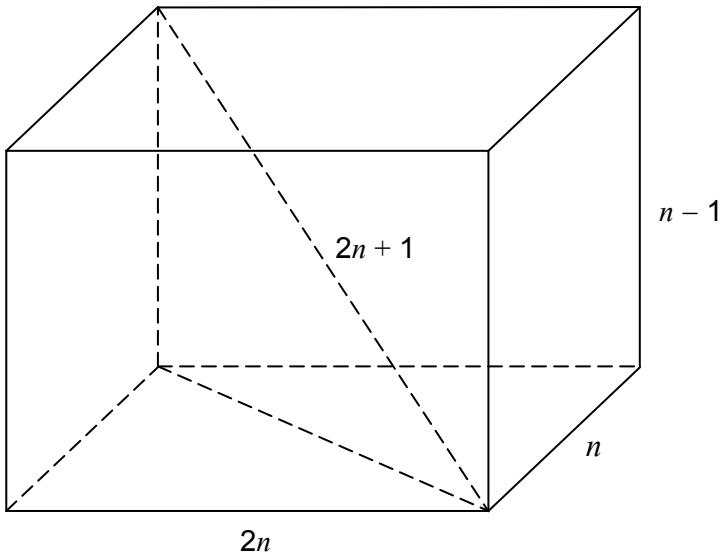
(c) $x + y : y$ *(2 marks)*

Question 10

A cuboid has dimensions $2n$, n and $n - 1$ cm.

A diagonal has length $2n + 1$ cm.

Not drawn accurately



Work out n. *(6 marks)*

AQA Qualifications

AQA Level 2 Certificate
FURTHER MATHEMATICS
Level 2 (8365)

Mark Scheme
Miscellaneous

Version 1.0

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M Miscellaneous

Question	Answer	Mark	Comments
1	$(x = 0, y = 8 \Rightarrow)\ a = 8$	M1	
	$1 = 8 \times b^{-3}$	M1	
	$b = 2$	A1	
	$y = 2^3 \times 2^x$	A1	must see both lines
	$= 2^{x+3}$		
2	$3 \times ? \times ? \times ?$	M1	
	$3 \times 4 \times 3 \times 2$ or 72	M1	
	$5 \times 4 \times 3 \times 2$ or 120	M1	oe
	192	A1	
3	LHS or LHS numerator	M1	Allow one error
	$6n^2 + 17n + 12 + 30n$		
	RHS $9n^2 + 12n$	M1	
	$0 = 3n^2 - 35n - 12$	M1	oe
	$(3n + 1)(n - 12)$	M1	
	12	A1	

Question	Answer	Mark	Comments
4	$4x^5 - 2x^3 - 2x^3 + x$	M1	At least two terms correct
	$4x^5 - 4x^3 + x = x + 4x^5 + 108$	M1	
	$4x^3 = -108$	M1	oe
	–3	A1	
5	$4a^3.5x$ and $6a^2.5^2x^2$	M2	M1 for each
	$4a^3.5 = 3 \times 6a^2.5^2$	M1	
	22.5	A1	

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Level 2 (8365)

Mark Scheme
Worksheet 1
Coordinate Geometry Circles

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1 Coordinate Geometry – Circles

Question	Answer	Mark	Comments
1(a)	$x^2 + (y - 3)^2 = 4$	B2	B1 LHS, B1 RHS
1(b)	$(x - 1)^2 + (y + 5)^2 = 16$	B2	B1 LHS, B1 RHS
1(c)	$(x + 3)^2 + (y - 4)^2 = 7$	B2	B1 LHS, B1 RHS
1(d)	$(x - 8)^2 + (y - 15)^2 = 289$ $(-8)^2 + (-15)^2$ $64 + 225 = 289$, Yes	B2 M1 A1	B1 LHS, B1 RHS oe
2(a)	$(r) = 6$ (centre =) (0, 0)	B2	B1 For each
2(b)	$(r) = 10$ (centre =) (3, 4)	B2	B1 For each
2(c)	$(r) = \sqrt{3}$ (centre =) (–5, 0)	B2	B1 For each
3	$\frac{-3 + 5}{2}$ or $\frac{6 + 12}{2}$ (1, 9) $\sqrt{(5 - 1)^2 + (12 - 9)^2}$ 5 $(x - 1)^2 + (y - 9)^2 = 25$	M1 A1 M1 A1 A1 ft	 oe ft Their centre ft Their centre and radius
4(a)	(3, 3)	B1	
4(b)	$\sqrt{2^2 + 1^2}$ $\sqrt{5}$ $(x - 1)^2 + (y - 2)^2 = 5$	M1 A1 B1 ft	oe ft Their radius

Question	Answer	Mark	Comments
5(a)	$\frac{12+14}{2}$ or $\frac{6+4}{2}$	M1	
	(13, 5)	A1	
5(b)	$\sqrt{(20-13)^2+(12-5)^2}$	M1	ft Their M
	$\sqrt{98}$	A1	$\sqrt{7^2+7^2}$
	$\sqrt{49\times 2} = 7\sqrt{2}$	A1	$\sqrt{7^2(1+1)} = 7\sqrt{2}$
5(c)	$\sqrt{(20-12)^2+(12-6)^2}$	M1	oe
	10	A1	
6	$\frac{-2+12}{2}$	M1	
	$\frac{0+4}{2}$	M1	
	C (2, 5)	A1	
7	Gradient $AC = \frac{6-3}{4--2}$	M1	oe
	$= \frac{3}{6}$ $\left(=\frac{1}{2}\right)$	A1	oe
	Gradient $BC = -2$	B1 ft	
	$\frac{6-k}{4-6} = -2$	M1	
	$k = 2$	A1	
8	$(13-5)^2+(-2-4)^2$	M1	
	$64 + 36 = 100$	A1	

Question	Answer	Mark	Comments
9	$(13-a)^2+(-2-4)^2=100$	M1	
	$169-13a-13a+a^2+36\ (=100)$	M1	Allow 1 error
	$a^2-26a+105=0$	A1	
	$(a-5)(a-21)=0$	M1	
	$a=5$ and $a=21$	A1	
10(a)	$\frac{3+11}{2}$	M1	oe eg, $3+4$
	$k=7$	A1	
10(b)	$\sqrt{6^2+(7-3)^2}$	M1	oe ft Their k
	$\sqrt{52}$	A1	
	$(x-6)^2+(y-7)^2=52$	A1 ft	ft Their k and their radius
11(a)	C is (3, 5)	B1	
	Gradient $CP=\frac{5-1}{3-4}$	M1	
	-4	A1	
	Gradient $OP=\frac{1}{4}$	B1	
	$-4\times\frac{1}{4}=-1$ So perpendicular (ie, tangent)	A1	
11(b)	$r=\sqrt{17}$	B1	
	$OP=\sqrt{4^2+1^2}$	M1	
	$=\sqrt{17}$	A1	

Question	Answer	Mark	Comments
12	Gradient $OP = \frac{2}{4}$ $\left(= \frac{1}{2}\right)$	B1	
	Gradient of tangent $= -2$	B1 ft	
	$y - 2 = -2\,(x - 4)$	M1	
	$y = -2x + 10$	A1	

13(a)	Centre (1, 9)	B2	B1 for each coordinate
	$r^2 = 3^2 + 4^2$ or $d^2 = 6^2 + 8^2$	M1	
	$(x-1)^2 + (y-9)^2 = 25$	A1ft	ft their centre
13(b)	Grad AB = $\frac{13-5}{4+2}$ or using their centre with A or B; or $\frac{8}{6}$ or $\frac{4}{3}$	M1	
	Grad tangent $-\frac{3}{4}$ or – their grad AB	M1	
	$y - 5 =$ their $-\frac{3}{4}(x+2)$	M1	
	$3x + 4y - 14 = 0$	A1	

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FURTHER MATHEMATICS
Level 2 (8360)

Mark Scheme
Worksheet 2
Geometric Problems and Proof

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2 Geometric Problems and Proof

Question	Answer	Mark	Comments
1	Let angle $SQR = x$	M1	Any order of angles
	$\therefore$ angle $RPQ = x$ alternate segment		
	$\therefore$ angle $RQP = x$ isosceles triangle	M1	
	$\therefore$ $\angle RQS = \angle RQP$	A1	SC2 'Correct' solution without reasons
2	Let angle $PSR = x$ = angle QRS	M1	$\angle PQR = 180 - x$
	$\therefore$ $\angle SPQ = 180 - x$ Allied angles on parallel lines		
	$\therefore$ $\angle SPQ + \angle QRS = 180$	A1	$\angle PSR + \angle PQR = 180$
	$PQRS$ is a cyclic quadrilateral (converse of) opposite angles add up to 180°	A1	SC2 'Correct' solution without reasons
3	$p + r = 180$	M1	
	$4x + 5x = 180$	M1	oe
	$(9x = 180)$ $x = 20$	A1	
	$6x = 120$	M1	ft Their x
	$s = 60$	A1 ft	ft Their x

Question	Answer	Mark	Comments
4	$\angle BED = x$ angles in same segment	M1	
	$\angle AEB = 90°$ angle in semicircle = 90°	A1	
	In ΔACE $y + x + 2x + x + 90 = 180$ angle sum of a triangle = 180	A1	
	$y + 4x = 180 - 90$ $= 90$	A1	SC2 'Correct' solution without reasons
5	$2x + 2y = 180$ opposite angles of a cyclic quadrilateral = 180	M1	
	$x + y = 90$	A1	
	$\therefore \angle QPS = 90$ angle sum of triangle = 180	A1	
	QS is diameter (converse of) angle in a semicircle = 90)	A1	SC2 'Correct' solution without reasons
6	Let $\angle SXT = x$ $\therefore \angle STX = x$ isosceles triangle	M1	
	$\therefore \angle SRT = x$ alternate segment	M1	
	$\therefore$ triangle RXT = is isosceles - 2 base angles equal	A1	SC2 'Correct' solution without reasons

Question	Answer	Mark	Comments
7	$\angle OAB = x$ isosceles triangle	M1	
	$\angle BOA = 180 - 2x$ angle sum of triangle = 180	M1	
	Reflex $\hat{BOA} = 360 - (180 - 2x)$	M1	
	(Angles at a point = 360) $= 180 + 2x$	A1	
	$y = 90 + x$ Angle at centre = 2 × angle at circumference	A1	SC3 'Correct' solution without reasons
8	$\angle QTP = x$ isosceles triangle	M1	
	$\angle VTR = x$ vertically opposite angles equal	M1	
	$\angle TQP = x = \angle RST$ exterior angle of cyclic quadrilateral = opposite interior angle	M1	oe
	$\therefore \angle VTR = \angle RST$ *PVT* is tangent (converse of) alternate segment theorem	A2	SC3 'Correct' solution without reasons
9	$\angle EDB = x$ alternate segment	M1	
	$\therefore \angle DCA = x$ corresponding angles equal	M1	
	$\therefore \angle DAB = x$ alternate segment	M1	
	ie, $\angle DAB = \angle EBF$	A2	SC3 'Correct' solution without reasons
	$\therefore$ *AD* is parallel to *BE* (converse of) corresponding angles equal		

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Level 2 (8360)

Mark Scheme
Worksheet 3
Algebraic Proof

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3 Algebraic Proof

Question	Answer	Mark	Comments
1	$4p - 12 - 4p + 2$ -10	M1 A1	4 terms with 3 correct
2	$8y + 24 + 6 - 3y$ or $5y + 30$ $5y + 30$ and $5(y + 6)$	M2 A1	M1 4 terms with 3 correct oe eg, $5y + 30$ and states both terms divisible by 5
3	$8a^3 + 4a^2 - 4a^2$ or $8a^3$ $8a^3$ and $(2a)^3$	M2 A1	M1 3 terms with 2 correct oe eg, $8a^3$ and states that 8 is a cube number
4	$a(x + 3)$ or $b(x + 3)$ $\frac{a(x+3)}{b(x+3)}$ and cancelling seen $\frac{a}{b}$ and explains that as numerator is smaller than denominator value will be < 1	M1 A1 A1	 oe
5(a)	$a = 3$ $b = 2$	B1 B1 ft	 ft 11 – their a^2
5(b)	$(x + 3)^2 \geq 0$ Adding 2 means always positive	M1 A1	oe Allow their a Must have $a = 3$ and $b = 2$

Question	Answer	Mark	Comments
6	$(x+1)^2$	B1	
	$(x+1)^2+5$	B1 ft	ft Their $(x+1)^2$
	$(x+1)^2 \geq 0$	M1	oe Allow their 1
	Adding 5 means always positive	A1	Must have $(x+1)^2+5$
7	$4x^2+6x+6x+9+8x+16$ or $4x^2+20x+25$	M2	M1 Allow one error in expansions
	$4x^2+20x+25$ and $(2x+5)^2$	A1	oe eg, $4x^2+20x+25$ and $(2x+5)(2x+5)$
	Explains that only solution is $(x=)-2.5$	A1	oe eg, explains that because the brackets are the same there is exactly one solution
8(a)	$\frac{1}{2}(n-1)(n-1+1)$	M1	
	$\frac{1}{2}n(n-1)$	A1	oe eg, $\frac{1}{2}n^2-\frac{1}{2}n$
8(b)	$\frac{1}{2}n(n+1)+\frac{1}{2}n(n-1)$	M1	$\frac{1}{2}n(n+1)$ + their (a)
	$\frac{1}{2}n^2+\frac{1}{2}n+\frac{1}{2}n^2-\frac{1}{2}n$	M1	Expands brackets ft Their (a)
	n^2	A1	
Alt 8(b)	$\frac{1}{2}n(n+1)+\frac{1}{2}(n+1)(n+1+1)$	M1	oe
	$\frac{1}{2}n^2+\frac{1}{2}n+\frac{1}{2}n^2+n+\frac{1}{2}n+1$	M1	Expands brackets oe eg, n^2+2n+1 ft Their $\frac{1}{2}(n+1)(n+1+1)$
	$(n+1)^2$	A1	

Question	Answer	Mark	Comments
9	$\frac{(x+2)(x-2)}{5(x-2)}$	M2	M1 For either numerator or denominator factorised correctly
	At least one correct cancellation in the product	M1	
	$2x^2$	A1	oe eg, $\frac{10x^2}{5}$
	Explains that 2 > 0 and $x^2 \geq 0$ so $2x^2$ always positive	A1	oe eg, Explains that 10 > 0 and 5 > 0 and $x^2 \geq 0$ so $\frac{10x^2}{5}$ always positive
10	$(3n)^2 - 3n + \{(n+1)^2 - (n+1)\}$	M1	oe $9n^2 - 3n$ or $n^2 + n + n + 1 - n - 1$
	$9n^2 - 3n + n^2 + n + n + 1 - n - 1$	A1	oe eg, $10n^2 - 2n$
	$10n^2 - 2n$ and $2n(5n-1)$	A1	oe eg $10n^2 - 2n$ and $k = 2$

AQA Qualifications

AQA Level 2 Certificate
FURTHER MATHEMATICS
Level 2 (8365)

Mark Scheme
Worksheet 4
Trigonometry

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4 Trigonometry

Question	Answer	Mark	Comments
1	$\sqrt{3}/2 + \sqrt{3}/2 - 1$	M1	Any 2 values correctly stated in surd form
	$\sqrt{3}/2 + \sqrt{3}/2 - 1$	M1	All 3 values correctly stated in surd form
	$\sqrt{3} - 1$	A1	
2	False	A1	
	True	A1	
	False	A1	
	True	A1	
3	Evidence that $\sin 45° = 1/\sqrt{2}$	B1	
	Area $= \frac{1}{2} \times 5 \times 6\sqrt{2} \times \sin 45°$	M1	
	15	A1	
4	$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$ seen	M1	
	$\frac{\sin^2\theta}{\cos^2\theta} \equiv \frac{1-\cos^2\theta}{\cos^2\theta}$	M1	
	$\tan\theta \equiv \frac{1}{\cos^2\theta} - 1$	A1	Accurate method with clear steps is required for all 3 marks
Alt 4	$\frac{1-\cos^2\theta}{\cos^2\theta}$	M1	oe
	$\frac{\sin^2\theta}{\cos^2\theta}$	M1	
	$\tan^2\theta$	A1	Accurate method with clear steps is required for all 3 marks

Question	Answer	Mark	Comments
5	Evidence that angle *ADC* is a right angle	M1	
	$\sin ACD = \frac{4}{5}$	M1	
	ACD = [53.1, 53.13010235]	A1	Allow 53 with method seen
	Angle ABD = [53.1, 53.13010235]	B1 ft	ft From 3rd mark their angle *ACD*
6	A triangle formed with *A*, *B* and the centre of the hemisphere with 2 sides of 10 cm and an angle of 120°	M1	
	$(AB^2 =)\ 10^2 + 10^2 - 2 \times 10 \times 10 \times \cos 120$	M1	$2 \times 10 \times \sin 60$
	(*AB* =) [17.3, 17.321]	A1	oe eg, $\sqrt{300}$
	$(\cos AOB =)\ \frac{30^2 + 30^2 - \text{their } AB^2}{2 \times 30 \times 30}$	M1	$2 \times \sin^{-1}$ (0.5 their *AB* ÷ 30)
	[33.557, 33.6]	A1 ft	ft Their *AB* Accept 34 with correct method seen
7	$\tan\theta = +\sqrt{2}$ or $\tan\theta = -\sqrt{2}$	M1	
	[54.7,54.74] or [125.26,125.3]	A1	
	180 + their [54.7,54.74] or 180 + their [125.26,125.3]	M1	
	[54.7,54.74] and [125.26,125.3] and 180 + their [54.7,54.74] and 180 + their [125.26,125.3]	A1ft	All 4 solutions [54.7,54.74] and [125.26,125.3] must be correct ft For other two solutions

Question	Answer	Mark	Comments
8	$(3\cos\theta - 1)(\cos\theta + 1)$	M2	M2 Fully correct use of quadratic formula M1 $(a\cos\theta + b)(c\cos\theta + d)$ where $ac = 3$ and $bd = \pm 1$ or quadratic formula with one sign error
	$\cos\theta = -1$ so $\theta = 180°$	A1	
	$\cos\theta = \frac{1}{3}$ so $\theta =$ [70.5, 70.53]	A1	
	$\theta = 289.5^0$	A1 ft	ft 360 – their [70.5, 70.53]
9 (a)	$\left(\frac{20}{2}\right)^2 + \left(\frac{30}{2}\right)^2$	M1	
	$\sqrt{325}$ or $5\sqrt{13}$ or 18.0(2…) or 18.03	A1	
(b)	$\frac{\sin CAB}{\text{their } 18.03} = \frac{\sin 59}{22}$	M1	oe
	$\sin^{-1}\left(\frac{\sin 59}{22} \times \text{their } 18.03\right)$	M1dep	
	44.6…	A1	

AQA Qualifications

AQA Level 2 Certificate
FURTHER MATHEMATICS
Level 2 (8360)

Mark Scheme
Worksheet 5
Matrices 1

Version 2.0

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5 Matrices 1

Question 1

Each question 2 marks. M1 for a correct row by column multiplication. A1 for the correct answer.

(a) $\begin{pmatrix} 30 \\ -16 \end{pmatrix}$ **(b)** $\begin{pmatrix} -15 \\ -20 \end{pmatrix}$ **(c)** $\begin{pmatrix} 10 & -4 \\ 12 & -6 \end{pmatrix}$

(d) $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ **(e)** $\begin{pmatrix} -24 & 42 \\ -6 & -18 \end{pmatrix}$ **(f)** $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Question 2

Each question 2 marks. M1 for a correct row by column multiplication. A1 for the correct answer.

(a) $\begin{pmatrix} -1 & 10 \\ 3 & -9 \end{pmatrix}$ **(b)** $\begin{pmatrix} 0 & -20 \\ 17 & 16 \end{pmatrix}$ **(c)** $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 34 & 19 \\ 2 & 60 \end{pmatrix}$ **(e)** $\begin{pmatrix} 0 & -5 \\ 1 & -11 \end{pmatrix}$ **(f)** $\begin{pmatrix} 11 & -19 \\ 13 & -22 \end{pmatrix}$

Question 3 (Non-calculator)

3 marks per question. 1 mark for multiplication of row by column, 1 mark for 2 simplified elements, 1 for other 2 elements correct. Part (c) 2 marks.

(a) $\begin{pmatrix} -1 & -2\sqrt{2} \\ -10\sqrt{2} & -12 \end{pmatrix}$ **(b)** $\begin{pmatrix} \frac{3}{2} & -5 \\ -\frac{11}{2} & 21 \end{pmatrix}$ **(c)** $\begin{pmatrix} 23 & 16 \\ 56 & 39 \end{pmatrix}$

(d) $\begin{pmatrix} 25 & 3\sqrt{3} \\ -10\sqrt{3} & 2 \end{pmatrix}$ **(e)** $\begin{pmatrix} \frac{7}{6} & 3 \\ \frac{19}{12} & 3 \end{pmatrix}$ **(f)** $\begin{pmatrix} 16 & 2\sqrt{2}+2\sqrt{3} \\ 7\sqrt{2}+7\sqrt{3} & 17 \end{pmatrix}$

Question 4

Each question 2 marks. M1 for a correct row by column multiplication. A1 for the correct answer.

(f) 3 marks. 2 for 1 pair correctly multiplied, 1 for final answer.

(a) $\begin{pmatrix} -p \\ -p-1 \end{pmatrix}$ **(b)** $\begin{pmatrix} 3x \\ 3y \end{pmatrix}$ **(c)** $\begin{pmatrix} 2m \\ m \end{pmatrix}$

(d) $\begin{pmatrix} -2a & 0 \\ 0 & 2a \end{pmatrix}$ **(e)** $\begin{pmatrix} 12t & 0 \\ 0 & 12t \end{pmatrix}$ **(f)** $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$

Question 5

(a) to (d) 2 marks each

(e) and (f) 3 marks each, 1 for a correct multiplication, 1 for two elements correct, 1 for all correct.

(a) $\begin{pmatrix} 2x^2+9 & 6x^2 \\ -17x & -15x \end{pmatrix}$ **(b)** $\begin{pmatrix} -23a & 41a \\ -24 & -5 \end{pmatrix}$ **(c)** $\begin{pmatrix} x^2 & 0 \\ 2x & x^2 \end{pmatrix}$

(d) $\begin{pmatrix} 2y & 3y^2+y \\ -6 & -9y+x \end{pmatrix}$ **(e)** $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ **(f)** $\begin{pmatrix} 9x^2+27 & -12x-3 \\ -36x-9 & x^2+2x+28 \end{pmatrix}$

AQA Qualifications

AQA Level 2 Certificate
FURTHER MATHEMATICS
Level 2 (8360)

Mark Scheme
Worksheet 6
Matrices 2

Version 2.0

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6 Matrices 2

Question 1

Each question 2 marks. M1 for a correct row by column multiplication. A1 for the correct answer.

(a) $\begin{pmatrix} 9 & 5 \\ 41 & 24 \end{pmatrix}$ **(b)** $\begin{pmatrix} -10 & 17 \\ -7 & 12 \end{pmatrix}$ **(c)** $\begin{pmatrix} 6 & -3 \\ 9 & 12 \end{pmatrix}$

(d) $\begin{pmatrix} 26 & 9 \\ 19 & 7 \end{pmatrix}$ **(e)** $\begin{pmatrix} 2 & -3 \\ -1 & 1 \end{pmatrix}$ **(f)** $\begin{pmatrix} -13 & 0 \\ -10 & 1 \end{pmatrix}$

Question 2

Each question 2 marks. M1 for a correct row by column multiplication. A1 for the correct answer.

(a) $\begin{pmatrix} 4 & 0 \\ -5 & 1 \end{pmatrix}$ **(b)** $\begin{pmatrix} 13 & 1 \\ -16 & -2 \end{pmatrix}$ **(c)** $\begin{pmatrix} -20 & 5 \\ 15 & -10 \end{pmatrix}$

(d) $\begin{pmatrix} -6 \\ 13 \end{pmatrix}$ **(e)** $\begin{pmatrix} -4 & 1 \\ 3 & -2 \end{pmatrix}$ **(f)** $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

Question 3

$-6 + 7a = 22$ M1

$a = 4$ A1

Question 4

Work out the values of a, b and c.

$$\begin{pmatrix} 2+2a & 6+ab \\ 5 & 9+b \end{pmatrix} = \begin{pmatrix} 12 & 26 \\ c & 13 \end{pmatrix}$$

$a = 5$, $b = 4$, $c = 5$ B1, B1, B1

Question 5

(4, 3) B2 (B1 for (4, ?), (?, 3) or $\begin{pmatrix}4\\3\end{pmatrix}$.

Question 6

$2m + 3n = -2$, $m + n = 0$ M1 for either, A1 for both

Attempt to solve M1

$m = 2$, $n = -2$ A1

Question 7

A = $\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$ B1

Rotation $\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}$ B1

Combined $\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$ M1 Multiplication in correct order.

A1 $\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$

Question 8

Reflection, in the line $y = -x$ B1, B1

Question 9 (Non-calculator)

Vertices of image A' $(h, 0)$ B' (h, h) $C'(0, h)$ Any 1 correct B1

Area of $OA'B'C'$. $= h2$ M1

$h = 3\sqrt{3}$ A1

Question 10

$BA = \begin{pmatrix} -3 & 0 \\ 0 & 3 \end{pmatrix}$ B1

$\begin{pmatrix} -3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 2 \\ 7 \end{pmatrix} = \begin{pmatrix} -6 \\ 21 \end{pmatrix}$ B1

Show this satisfies $7x + 2y = 0$ M1

AQA Qualifications

AQA Level 2 Certificate
FURTHER MATHEMATICS
Level 2 (8360)

Mark Scheme
Worksheet 7
Inequalities

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7 Inequalities

Question	Answer	Mark	Comments
1	$-2 < x \leq 2$ $-1 \quad 0 \quad 1 \quad 2$	M1 A2	 A1 3 correct with none incorrect or 4 correct with one incorrect
2	$6x + 2x > 24$ $x > 3$	M1 A1	oe
3	$8x - 4 < 2$ $8x < 2 + 4$ $x < \frac{3}{4}$	M1 M1 A1	oe $2x - 1 < \frac{2}{4}$ oe oe $2x < \frac{2}{4} + 1$ oe
4	$4(2y + 6) > 2y + 10 + 2y + 10 + y + 4 + y + 4$ $8y - 6y > 28 - 24$ $y > 2$ or $k = 2$	M2 M1 A1	oe eg, $8y + 24 > 6y + 28$ M1 $4(2y + 6)$ or $2y + 10 + 2y + 10 + y + 4 + y + 4$ oe
5	Always Never Sometimes Sometimes	B4	B1 For each correct part

Question	Answer	Mark	Comments
6(a)	(4, 0)	B1	
	(–4, 0)	B1	SC1 4 and –4 seen
6(b)	$-4 \leq x \leq 4$	B2 ft	ft Their 4 and their –4 B1 ft $-4 < x < 4$
Alt 6(b)	$(4 + x)(4 - x)$ and –4 and 4	M1	
	$-4 \leq x \leq 4$	A1	
7(a)	$x(x + 3)$	B1	
7(b)	U-shaped parabola	M1	
	0 and – 3 labelled on x-axis	A1 ft	ft Their factors in (a)
7(c)	$x < -3$ and $x > 0$	B2 ft	ft Their factors in (a) B1 ft $x \leq -3$ and $x \geq 0$
8	5 and –2	B1	
	Sketch of graph $y = (x - 5)(x + 2)$	M1	Sign diagram using their 5 and their –2
	$x < -2$ and $x > 5$	A1	
9	$(x + 6)(x - 2)$	M1	$(x + a)(x + b)$ where $ab = \pm 12$ or $a + b = \pm 4$
	–6 and 2	A1	
	Sketch of graph $y = (x + 6)(x - 2)$	M1	Sign diagram using their –6 and their 2
	$-6 < x < 2$	A1	

Question	Answer	Mark	Comments
10	$(2x-3)(x+1)$	M1	$(2x+a)(x+b)$ where $ab=\pm 3$ or $a+2b=\pm 1$
	$\frac{3}{2}$ and -1	A1	oe
	Sketch of graph $y=(2x-3)(x+1)$	M1	Sign diagram using their $\frac{3}{2}$ and their -1
	$-1<x<\frac{3}{2}$	A1	
11	$(3x-2)(x-4)$	M1	$(3x+a)(x+b)$ where $ab=\pm 8$ or $a+3b=\pm 14$
	$\frac{2}{3}$ and 4	A1	
	Sketch of graph $y=(3x-2)(x-4)$	M1	Sign diagram using their $\frac{2}{3}$ and their 4
	$x<\frac{2}{3}$ and $x>4$	A1	
12	$n^2>\frac{1}{2}(4n-8)n$	M1	oe
	$0>n^2-4n$	A1	
	$n(n-4)$	M1	Factorises their quadratic expression
	Sketch of graph of $y=n(n-4)$	M1	Sign diagram using their 0 and their 4
	$0<n<4$	A1	

AQA Qualifications

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FURTHER MATHEMATICS
Level 2 (8365)

Mark Scheme
Worksheet 8
Functions

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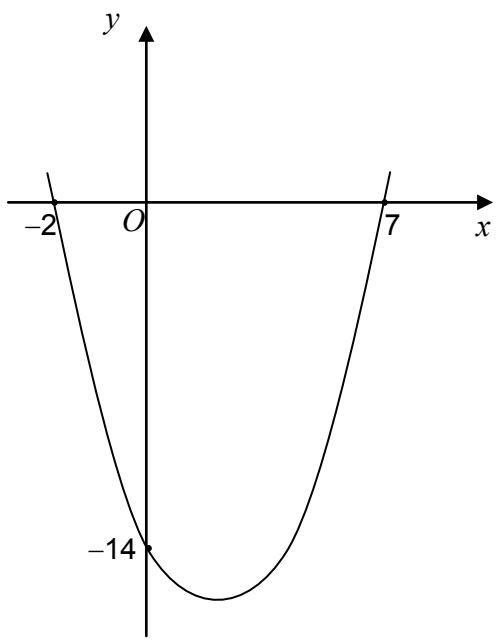
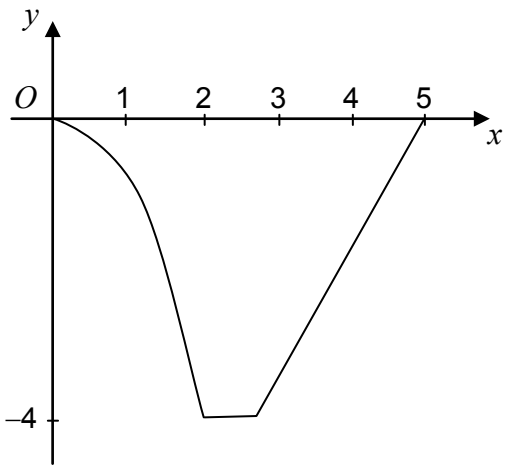
8 Functions

Question	Answer	Mark	Comments
1	$2x^3 - 250 = 0$ $x^3 = \frac{250}{2}$ $x = 5$	M1 M1 A1	 oe
2	$(-3)^2 + a(-3) - 8 = 13$ $9 - 8 - 13 = 3a$ $a = -4$	M1 M1 A1	 oe Allow 1 error
3	$(x + 2)^2 + 3(x + 2) - 10$ $x^2 + 2x + 2x + 4 + 3x + 6 - 10$ $x^2 + 7x$ $= x(x + 7)$	M1 M1 A1	 oe Allow 1 error
4(a)	f(x) … 6	B1	
4(b)	$-11 \leqslant$ f(x) $\leqslant 13$	B1	B1 For –11 or 13 seen
4(c)	f(x) > 48	B1	
5(a)	Not defined when $x = 3$ or cannot divide by 0 when $x = 3$	B1	oe
5(b)	x … a where a … 5 or $x > a$ where a … 5	B1	eg x … 5 $x > 6$ Allow list of x values if all are … 5

Question	Answer	Mark	Comments
6	Either $3 - 2x = -5$ or $3 - 2x = 5$	M1	
	$a = -1$	A1	
	$b = 4$	A1	SC2 $a = 4$, $b = -1$
7	Attempt to complete the square in the form $(x + 3)^2$	M1	
	$(x + 3)^2 - 9 + a$	A1	oe
	$a = 20$	A1	
8(a)	$(x + a)(x + b)$	M1	$ab = -14$ or $a + b = -5$
	$(x - 7)(x + 2)$	A1	
8(b)		B3	B1 Curve through their (7, 0) and (–2, 0) (from 8(a)) B1 Curve through (0, –14) B1 Smooth U shape

Question	Answer	Mark	Comments
9		B3	B1 For each part
10	(3, 0) and (7, 0) marked or used (1, 2) and (4, –1) marked or used Either of their triangular areas calculated correctly $\frac{1}{2} \times 3 \times 2$ and $\frac{1}{2} \times 4 \times 1$ $= 3 : 2$	M1 M1 M1 A1	
11	$2y + a = \sqrt{x}$ $f^{-1}(x) = (2x + a)^2$ $f^{-1}(3a) = (7a)^2$ or $49a^2 = 306.25$ 2.5	M1 M1 M1 A1	oe oe oe
12	$\frac{2\left(\frac{5}{x+1}\right)-1}{4}$ $\frac{9-x}{4x+4}$ or $\frac{-x+9}{4x+4}$	M1 A1	
13	$-\frac{1}{2} < x < 5$	B2	B1 for stating must be negative or correct inequality with one or two ≤ symbols in place of <

AQA Qualifications

AQA Level 2 Certificate
FURTHER MATHEMATICS
Level 2 (8360)

Mark Scheme
Worksheet 9
Coordinate Geometry - Calculus

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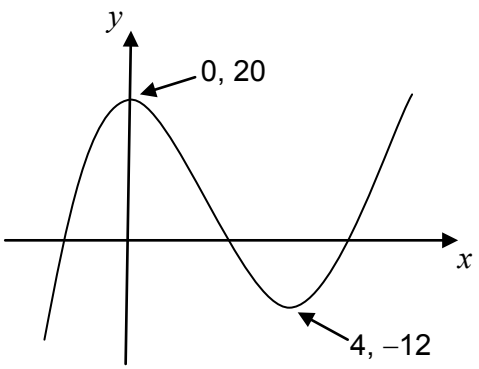
9 Coordinate Geometry – Calculus

Question	Answer	Mark	Comments
1(a)	5 $-\frac{1}{5}$ –4	B1 B1 ft B1	 ft $\frac{-1}{\text{their } 5}$
1(b)	–2 $\frac{1}{2}$ 3	B1 B1 ft B1	 ft $\frac{-1}{\text{their } -2}$
1(c)	$\frac{2}{3}$ $-\frac{3}{2}$ 4	B1 B1 ft B1	 ft $\frac{-1}{\text{their } \frac{2}{3}}$
1(d)	$\frac{5}{2}$ $-\frac{2}{5}$ $\frac{15}{2}$	B1 B1 ft B1	 ft $\frac{-1}{\text{their } \frac{5}{2}}$
1(e)	$\frac{3}{4}$ $-\frac{4}{3}$ –6	B1 B1 ft B1	 ft $\frac{-1}{\text{their } \frac{3}{4}}$

Question	Answer	Mark	Comments
2(a)	$\left(\frac{1}{2}, -\frac{1}{2}\right)$	B2	B1 For each coordinate
	1	B1	
	$\sqrt{(7^2 + 7^2)}$	M1	
	$\sqrt{98}$ or $7\sqrt{2}$	A1	
2(b)	$(-1\frac{1}{2}, 3)$	B2	B1 For each coordinate
	$\frac{4}{5}$	B1	
	$\sqrt{(5^2 + 4^2)}$	M1	
	$\sqrt{41}$	A1	
2(c)	$(2\frac{1}{2}, 4)$	B2	B1 For each coordinate
	$-\frac{12}{5}$	B1	
	$\sqrt{(5^2 + 12^2)}$	M1	
	13	A1	
2(d)	$(-4, -3)$	B2	B1 For each coordinate
	$-\frac{3}{2}$	B1	
	$\sqrt{(4^2 + 6^2)}$	M1	
	$\sqrt{52}$ or $2\sqrt{13}$	A1	
2(e)	$(5, 1\frac{1}{2})$	B2	B1 For each coordinate
	$-\frac{15}{8}$	B1	
	$\sqrt{(8^2 + 15^2)}$	M1	
	17	A1	

Question	Answer	Mark	Comments
2(f)	(1, –1)	B2	B1 For each coordinate
	$\frac{1}{3}$	B1	
	$\sqrt{(12^2 + 4^2)}$	M1	
	$\sqrt{160}$ or $4\sqrt{10}$	A1	
3(a)	(5, –3)	B2	B1 For each coordinate
3(b)	(4, –6)	B2	B1 For each coordinate
3(c)	(–5, –8)	B2	B1 For each coordinate
3(d)	(9, 7)	B2	B1 For each coordinate
3(e)	(–7, 9)	B2	B1 For each coordinate
4	$x^2 + 7 = 5x + 1$ or $x^2 - 5x + 6 = 0$	M1	
	$(x - 2)(x - 3) = 0$	M1	Attempt to factorise the quadratic
	(2, 11) or (3, 16)	A1 ft	ft Their factors
	(2, 11) and (3, 16)	A1	
5	Gradient of $L = -3$	B1	
	Gradient of $N = \frac{1}{3}$	M1	
	$y - (-1) = \frac{1}{3}(x - 3)$	M1	
	$y = \frac{1}{3}x - 2$	A1	

Question	Answer	Mark	Comments
6(a)	$\frac{dy}{dx} = 7$	B1	
6(b)	$\frac{dy}{dx} = 2x - 5$	B2	B1 For each term
6(c)	$\frac{dy}{dx} = 9x^2 + 4$	B2	B1 For each term
6(d)	$\frac{dy}{dx} = 3x^2 - 14x + 10$	B2	B1 For two terms correct
6(e)	$y = 4x^3 + 8x^2 - 12x$ $\frac{dy}{dx} = 12x^2 + 16x - 12$	B1 B2 ft	B1 For two terms correct ft Their $y =$ ....
6(f)	$y = 3x^2 + 19x - 40$ $\frac{dy}{dx} = 6x + 19$	B1 B2 ft	B1 For one term correct ft Their $y =$ ....
6(g)	$y = 42x - 20x^2 + 2x^3$ $\frac{dy}{dx} = 42 - 40x + 6x^2$	B1 B2 ft	B1 For two terms correct ft Their $y =$ ….
6(h)	$y = x^3 - 4x^2 - 15x + 18$ $\frac{dy}{dx} = 3x^2 - 8x - 15x$	B2 B2 ft	B1 For four terms, three of which are correct B1 For two terms correct ft Their $y =$ ....
7	$\frac{dy}{dx} = 3x^2 + 2x + 2$ (when $x = -2$) gradient tgt = 10 (when $x = -2$) $y = -12$ $y - (-12) = 10(x - (-2))$ $y = 10x + 8$	M1 A1 B1 M1 A1 ft	 oe ft Their m and c

Question	Answer	Mark	Comments
8	$\frac{dy}{dx} = 3x^2 + 4x - 9$	M1	
	(when $x = 1$) gradient tgt $= -2$	A1	
	(when $x = 1$) gradient nl $= \frac{1}{2}$	A1 ft	ft Their -2
	$y - (-3) = \frac{1}{2}(x - 1)$	M1	oe
	$x - 2y - 7 = 0$	A1ft	ft Their m and c
9(a)	$\frac{dy}{dx} = 3x^2 - 12x$	M1	
9(b)	$3x^2 - 12x = 0$ or $3x(x - 4) = 0$	M1	
	$x = 0$ and $x = 4$	A1	
	(0, 20) and (4, -12)	A1	
	Testing the sign of $\frac{dy}{dx}$ for values of x either side of 0 and 4	M1	
	Maximum at (0, 20) Minimum at (4, -12)	A1	If previous M1 earned
9(c)		B2	B1 For correct general shape B1 ft For labelling the stationary points

Question	Answer	Mark	Comments
10(a)	$\frac{dy}{dx} = 3x^2 - 2x + k$	B1	
10(b)	$3(2)^2 - 2(2) + k = 0$ $k = -8$	M1 A1	
10(c)	$3x^2 - 2x - 8 = 0$ $(3x + 4)(x - 2) = 0$ Maximum at $x = -\frac{4}{3}$	M1 A1 A1	
11(a)	$\frac{dy}{dx} = \frac{1}{2}x - 1$ (when $x = 3$) $\frac{dy}{dx} = \frac{3}{2} - 1 = \frac{1}{2}$ $y - (-\frac{3}{4}) = \frac{1}{2}(x - 3)$ $y = \frac{1}{2}x - 1\frac{1}{2} - \frac{3}{4}$	M1 A1 M1 A1	Clearly shown since $y = \frac{1}{2}x - \frac{9}{4}$ answer given
11(b)	Gradient tangent at $B = -2$ $\frac{1}{2}x - 1 = -2$ $x = -2$ $B = (-2, 3)$	B1 M1 A1 ft A1	ft Their tangent gradient
12(a)	$-6x^{-3}$	B1	
12(b)	$-5x^{-2} + 4x$	B2	B1 for each term
12(c)	$-9x^{-4} + 20x^{-6}$	B2	B1 for each term
12(d)	$-10x^{-3} - x^{-2}$	B2	B1 for each term
12(e)	$x^3 + 2 - 4x^{-1}$ $3x^2 + 4x^{-2}$	B1 B2ft	B1ft for each term

12(f)	$\frac{3}{4}x^{-2} + \frac{1}{2}x^{3}$ $-\frac{3}{2}x^{-3} + \frac{3}{2}x^{2}$	B1 B2ft	 B1ft for each term

13(a)	Hypotenuse $10x$ $2y = 84 - 36x$ $y = 42 - 18x$	M1 A1	
13(b)	$A = 16x(42 - 18x) + ½ \times 16x \times 6x$ $A = 672x - 288x^{2} + 48x^{2}$ $= 672x - 240x^{2}$	M1 A1	
13(c)	$\frac{dA}{dx} = 672 - 480x$ $= 0$ when $x = \frac{672}{480}$ or 1.4 470.4	M1 M1 A1	

14	$\frac{x}{4} + 8x^{-2}$ or $\frac{dy}{dx} = \frac{1}{4}$ .....seen $\frac{dy}{dx} = \frac{1}{4} - \frac{16}{x^{3}}$ $= 0$ when $\frac{1}{4} = \frac{16}{x^{3}}$ or $x^{3} = 64$ or $x = 4$ 1.5	M1 M1 M1 A1	 oe

AQA Qualifications

AQA Level 2 Certificate
FURTHER MATHEMATICS
Level 2 (8365)

Mark Scheme
Worksheet 10
Factor Theorem

Version 2.0

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10 Factor Theorem

Question	Answer	Mark	Comments
1(a)	$x(x^2 - 5x - 36)$	B1	
1(b)	$x = 0,\ x = -4,\ x = 9$	B2	B1 For two solutions
2(a)	$f(1) = 1 + 2 - 5 - 6 = -8$ $f(-1) = -1 + 2 + 5 - 6 = 0$	B1 B1	
2(b)	$f(2) = 8 + 8 - 10 - 6 = 0$ $f(-2) = -8 + 8 + 10 - 6 = 4$	B1 B1	
2(c)	$f(3) = 27 + 18 - 15 - 6 = 24$ $f(-3) = -27 + 18 + 15 - 6 = 0$	B1 B1	
2(d)	$(x + 1)$, $(x - 2)$ and $(x + 3)$	B1	

Question	Answer	Mark	Comments
3(a)	$(-5)^3 + 7(-5)^2 + 2(-5) - 40$	M1	oe
	$-125 + 175 - 10 - 40 = 0$	A1	Clearly shown to = 0
3(b)	$x^3 + 7x^2 + 2x - 40$ $\equiv (x + 5)(x^2 + kx - 8)$	M1	Sight of x^2 and -8 in a quadratic factor
	$(x - 2)$	A1	
	$(x + 4)$	A1	
Alt 1 **3(b)**	Substitutes another value into the expression and tests for '= 0'	M1	
	$(x - 2)$	A1	
	$(x + 4)$	A1	
Alt 2 **3(b)**	Long division of polynomials getting as far as $x^2 + 2x$	M1	
	$(x - 2)$	A1	
	$(x + 4)$	A1	
3(c)	$(x =) -5, -4$ and 2	B1	

4	$(-2)^3 + 5(-2)^2 + 9(-2) + k = 0$	M1	
	$-8 + 20 - 18 + k = 0$	A1	
	$k = 6$	A1	

Question	Answer	Mark	Comments
5(a)	$(-3)^3 + (-3)^2 + (-3)a - 72 = 0$	M1	
	$-27 + 9 - 3a - 72 = 0$	A1	
	$a = -30$	A1	
5(b)	$x^3 + x^2 - 30x - 72$ $\equiv (x + 3)(x^2 + kx - 24)$	M1	Sight of x^2 and -24 in a quadratic factor
	$(x + 4)$	A1	
	$(x - 6)$	A1	
Alt 1 5(b)	Substitutes another value into the expression and tests for '$= 0$'	M1	
	$(x + 4)$	A1	
	$(x - 6)$	A1	
Alt 2 5(b)	Long division of polynomials getting as far as $x^2 - 2x$	M1	
	$(x + 4)$	A1	
	$(x - 6)$	A1	

Question	Answer	Mark	Comments
6(a)	$(x-3)(x+4)(x+k)$ $\equiv x^3+ax^2+bx+24$	M1	or $-3\times4\times k=24$
	$(x-2)$	A1	
6(b)	$(x-3)(x+4)(x-2)$	M1	
	$(x-3)(x^2+2x-8)$	M1	oe
	$x^3-x^2-14x+24$	A1	
	$a=-1$ and $b=-14$	A1 ft	ft Their expansion
Alt 6(b)	Substitutes any two of $x=-4$, $x=2$ or $x=3$ into $x^3+ax^2+bx+24$ to create simultaneous equations	M1	
	Any two of $-64+16a-4b+24=0$ or $8+4a+2b+24=0$ or $27+9a+3b+24=0$	M1	
	$a=-1$	A1	
	$b=-14$	A1 ft	ft Their first solution

Question	Answer	Mark	Comments
7(a)	$(5)^3 + k(5)^2 + 9(5) - 20 = 0$	M1	
	$125 + 25k + 45 - 20 = 0$	A1	
	$k = -6$	A1	
7(b)	$x^3 - 6x^2 + 9x - 20$ $\equiv (x - 5)(x^2 + kx + 4)$	M1	Sight of x^2 and 4 in a quadratic factor
	$(x - 5)(x^2 - x + 4)$	A1	
7c	Tests '$b^2 - 4ac$' for the quadratic	M1	ft Their quadratic or attempts to solve their quadratic = 0
	Shows '$b^2 - 4ac$' = −15 (or < 0) and states no more linear factors	A1	States 'no solutions' to their quadratic = 0
8	Substitutes a value of x into the expression and tests for '= 0'	M1	
	Works out first linear factor $(x + 1)$, $(x + 2)$ or $(x - 9)$	A1	
	$x^3 - 6x^2 - 25x - 18$ $\equiv (x + 1)(x^2 + kx - 18)$ or $(x + 2)(x^2 + kx - 9)$ or $(x - 9)(x^2 + kx + 2)$	M1	Attempts to work out the quadratic factor Sight of x^2 and −18 in a quadratic factor or sight of x^2 and −9 in a quadratic factor or sight of x^2 and 2 in a quadratic factor
	2nd and 3rd linear factors	A1	
	−1, −2 and 9	A1	
Alt 1 **8**	Substitutes a value of x into the expression and tests for '= 0'	M1	
	Works out first linear factor $(x + 1)$, $(x + 2)$ or $(x - 9)$	A1	
	Substitutes another value into the expression and tests for '= 0'	M1	
	2nd and 3rd linear factors	A1	
	−1, −2 and 9	A1	

Question	Answer	Mark	Comments
Alt 2 8	Substitutes a value of x into the expression and tests for '= 0'	M1	
	Works out first linear factor $(x + 1)$, $(x + 2)$ or $(x - 9)$	A1	
	Long division of polynomials getting as far as $x^2 - 7x$ or $x^2 - 8x$ or $x^2 + 3x$	M1	Depending on first linear factor
	2nd and 3rd linear factors	A1	
	–1, –2 and 9	A1	
9(a)	32 – 32 – 162 + 162	B1	
9(b)	$(x – 2)(x^4 – 81)$	M1	
	$(x – 2)(x^2 + 9)(x^2 – 9)$	M1	
	$(x – 2)(x^2 + 9)(x + 3)(x – 3)$	M1	
	2, –3 and 3	A1	
10(a)	$3\left(-\frac{2}{3}\right)^3 + 2\left(-\frac{2}{3}\right)^2 - 3\left(-\frac{2}{3}\right) - 2$	M1	Oe
	$-\frac{8}{9} + \frac{8}{9} + 2 – 2 = 0$	A1	Clearly shown to = 0
10(b)	$(3x + 2)(x^2 – 1)$	M1	
	$(3x + 2)(x + 1)(x – 1)$	A1	

AQA Qualifications

AQA Level 2 Certificate
FURTHER MATHEMATICS
Level 2 (8360)

Mark Scheme
Worksheet 11
Sequences

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11 Sequences

For the nth terms of quadratic sequences two methods are shown (see example 2).
Other valid methods may be used.

Question	Answer	Mark	Comments
1	$-4n$	M1	
	$254 - 4n$	A1	
	$254 - 4n < 0$	M1	oe
	64th	A1	
2	**Method A**	M1	
	8 9 14 23 36		
	1 5 9 13		
	4 4 4		
	Subtract $2n^2$ from sequence	A1	
	6 1 -4 ……		
	nth term of this sequence is	M1	
	11 $-5n$		
	Giving $2n^2 - 5n + 11$	A1	
Alt 2	**Method B**	M1	
	Using $an^2 + bn + c$		
	$a + b + c = 8$		
	$4a + 2b + c = 9$		
	$9a + 3b + c = 14$		
	$3a + b = 1$	M1	oe
	$5a + b = 5$		
	$a = 2$ and $b = -5$	A1	
	Giving $2n^2 - 5n + 11$	A1	

Question	Answer	Mark	Comments
3(a)	Use Method A or B from Q2	3 marks	or any other valid method
3(b)(i)	$n^2 + 3n + 1$	B1	
3(b)(ii)	$n^2 + 4n$	B1	
4(a)	$3n + 1$	B1	
4(b)	$(3n + 1)^2$	B1	oe
4(c)	$49 \times 169 = 7^2 \times 13^2$ 30th is 91^2 $= (7 \times 13)^2$ $= 7^2 \times 13^2$	B1 M1 A1	oe 8281 oe 8281
5	nth term of lengths is $n + 3$ nth term of widths is $n + 2$ Area is $(n + 3)(n + 2)$ $n^2 + 3n + 2n + 6$ $= n^2 + 5n + 6$	M1 M1 M1 A1	
Alt 5	nth term of 12 20 30 by Method A or Method B	4 marks	or any other valid method
6(a)	$a + 9b = 35$ $a + 15b = 59$ $6b = 24$ $b = 4$ $a = -1$	M1 M1 A1 A1 ft	 oe
6(b)	3 11 19 … … $8n - 5$	B1 ft B1 ft	

Question	Answer	Mark	Comments
7(a)	$\frac{3n+1}{n} - \frac{3(n+1)+1}{n+1}$ $\frac{(3n+1)(n+1) - n(3n+4)}{n(n+1)}$ $\frac{3n^2+n+3n+1-3n^2-4n}{n(n+1)}$ $= \frac{1}{n(n+1)}$	M1 M1 A1	oe eg subtracts in different order oe
Alt 7(a)	$\frac{3n+1}{n} = 3 + \frac{1}{n}$ $(3 + \frac{1}{n}) - (3 + \frac{1}{n+1})$ $\frac{n+1-n}{n(n+1)}$ $= \frac{1}{n(n+1)}$	M1 M1 A1	oe eg subtracts in different order oe
7(b)	Any substitution and evaluation for $1 \leqslant n \leqslant 10$ eg $\frac{1}{9 \times 10} = \frac{1}{90}$ or $\frac{1}{10 \times 11} = \frac{1}{110}$ 10th and 11th	M1 A1	oe eg $1 < 0.01n^2 + 0.01n$ and attempt to solve
7(c)	3	B1	

Question	Answer	Mark	Comments
8	$\frac{5n+2}{2n} = \frac{5n}{2n} + \frac{2}{2n}$ $\left(\frac{5}{2} + \frac{1}{n}\right)$ $\frac{1}{n} \rightarrow 0$ as $n \rightarrow \infty$ $S = \frac{5}{2}$ $(= 2.5)$	M1 A1	oe
9	Odd number is $2n + 1$ **or** $2n - 1$ $2n - 1$ **and** $2n + 1$ Sequence is $(2n - 1)(2n + 1)$ $(= 4n^2 - 1)$	M1 M1 A1	
Alt 9	Using Method A or Method B giving $4n^2 - 1$	3 marks	or any other valid method eg 1 4 9 16 → n^2 4 16 36 64 → $4n^2$ 3 15 35 63 → $4n^2 - 1$
10(a)	$T_1 = \frac{1}{5}$ $T_2 = \frac{7}{14}$ $(= \frac{1}{2})$ $\frac{5}{10} - \frac{2}{10} = \frac{3}{10}$	B1 B1 B1	 oe oe
10(b)	$\frac{2}{3}$	B1	

AQA Qualifications

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FURTHER MATHEMATICS
Level 2 (8360)

Mark Scheme
Worksheet 12
Algebraic Problems – including ratio

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12 Algebraic Problems – including ratio

Question	Answer	Mark	Comments
1	$n = 4$	M1	
	$\frac{1}{2}$	A1	
	$n = -4$	M1	
	$\frac{9}{10}$	A1	
2(a)	$\frac{x}{y} = \frac{6}{5}$	M1	
	$x = \frac{6y}{5}$	A1	oe
2(b)	$\frac{6y}{5} + \frac{15y}{5} : \frac{12y}{5} - \frac{5y}{5}$	M1	oe $6 + 3 \times 5 : 2 \times 6 - 5$
	$\frac{21(y)}{(5)} : \frac{7(y)}{(5)}$	A1	
3	$\frac{3}{10}$ of $(6a - a)$ **or** $\frac{3}{10}$ of $(11b - b)$	M1	oe
	$(2.5a, 4b)$	A2	oe A1 For each coordinate SC2 $(1.5a, 3b)$
4	$\frac{a + 3b}{a + 7b} = \frac{2}{5}$	M1	
	$5a + 15b = 2a + 14b$	M1	Allow one error
	$3a + b = 0$	A1	oe
	$a + b = -4$ $2a = 4$	A1 ft	
	$a = 2$ and $b = -6$	A1 ft	

Question	Answer	Mark	Comments
5	$\frac{a}{b} = \frac{4}{3}$	M1	oe
	$b = \frac{3a}{4}$	A1	$a = \frac{4b}{3}$
	$a \times \frac{3a}{4} + a = 5$	M1	$\frac{4b}{3} \times b + \frac{4b}{3} = 5$
	$3a^2 + 4a - 20 = 0$	A1	$4b^2 + 4b - 15 = 0$
	$(3a + 10)(a - 2) = 0$	M1	$(2b + 5)(2b - 3)$
	$a = -\frac{10}{3}$ $a = 2$	A1 ft	$b = -\frac{5}{2}$ $b = \frac{3}{2}$
	$b = -\frac{5}{2}$ $b = \frac{3}{2}$	A1 ft	$a = -\frac{10}{3}$ $a = 2$
6	Let their ages 6 years ago be $8x$ and $5x$	M1	
	$8x + 5x = 90 - 12$	M1	Allow $90 - 6$ for M1
	$13x = 78$ $(x = 6)$	A1	
	Their 6×8 and their 6×5 (48) (30)	M1	
	54 and 36	A1	
Alt 6	$x + y = 90$	M1	
	$\frac{x - 6}{y - 6} = \frac{8}{5}$	M1	
	$18 = 8y - 5x$	A1	
	Eliminates a letter	M1	
	$(x =)$ 54 and $(y =)$ 36	A1	

Question	Answer	Mark	Comments
7	x, x and $180 - 2x$ seen or on diagram	M1	
	$\frac{x}{y} = \frac{4}{5}$	M1	
	$x = \frac{4y}{5}$	A1	oe
	$2y = 180 - 2x$ (or $y = 90 - x$)	M1	oe
	$y = 90 - \frac{4y}{5}$	M1	oe
	$\frac{9y}{5} = 90$	M1	oe
	$y = 50$	A1	
8	$a = 7x + 18$ or $b = 3x + 18$	B1	oe
	$\frac{\text{their}(7x + 18)}{\text{their}(3x + 18)} = \frac{3}{2}$	M1	
	$14x + 36 = 9x + 54$	M1	Rearranging
	$5x = 18$	M1	Solving
	$x = 3.6$	A1	
9(a)	$x : y = 6 : 10$	M1	oe
	$x : y : z = 6 : 10 : 9$	M1	
	$x : z = 2 : 3$	A1	
9(b)	$3 \times 10 : 7 \times 5$	M1	oe
	$6 : 7$	A1	
9(c)	$3 + 5 : 5$	M1	$\frac{x + y}{y} = \frac{x}{y} + 1$ or $\frac{3}{5} + 1$
	$8 : 5$	A1	

Question	Answer	Mark	Comments
10	$(2n)^2 + n^2$	M1	oe
	$(2n)^2 + n^2 + (n-1)^2 = (2n+1)^2$	M1	
	$4n^2 + n^2 + n^2 - n - n + 1$ $= 4n^2 + 2n + 2n + 1$	M1	Allow one error
	$2n^2 - 6n = 0$	M1	Rearranging ; or $2n^2 = 6n$
	$2n(n-3) = 0$	M1	(allow ÷ by n) $2n = 6$
	$n = 3$	A1	