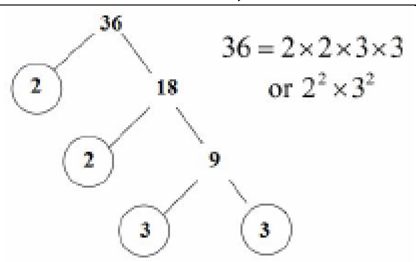
Topic: Basic Number and Decimals

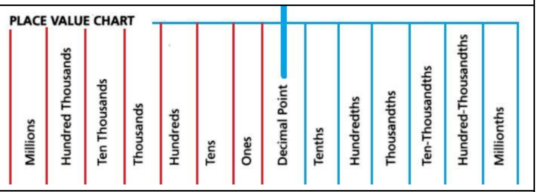
Topic/Skill	Definition/Tips	Example
1. Integer	A **whole number** that can be positive, negative or zero.	$-3, 0, 92$
2. Decimal	A number with a **decimal point** in it. Can be positive or negative.	$3.7, 0.94, -24.07$
3. Negative Number	A number that is **less than zero**. Can be decimals.	$-8, -2.5$
4. Addition	To find the **total**, or **sum**, of two or more numbers. 'add', 'plus', 'sum'	$3 + 2 + 7 = 12$
5. Subtraction	To find the **difference** between two numbers. To find out how many are left when some are taken away. 'minus', 'take away', 'subtract'	$10 - 3 = 7$
6. Multiplication	Can be thought of as **repeated addition**. 'multiply', 'times', 'product'	$3 \times 6 = 6 + 6 + 6 = 18$
7. Division	Splitting into equal parts or groups. The process of calculating the **number of times one number is contained within another one**. 'divide', 'share'	$20 \div 4 = 5$ $\frac{20}{4} = 5$
8. Remainder	The amount '**left over**' after dividing one integer by another.	The remainder of $20 \div 6$ is 2, because 6 divides into 20 exactly 3 times, with 2 left over.
9. BIDMAS	An acronym for the **order** you should do calculations in. BIDMAS stands for **'Brackets, Indices, Division, Multiplication, Addition and Subtraction'**. Indices are also known as 'powers' or 'orders'. With strings of division and multiplication, or strings of addition and subtraction, and no brackets, work from left to right.	$6 + 3 \times 5 = 21, not\ 45$ $5^2 = 25$, where the 2 is the index/power. $12 \div 4 \div 2 = 1.5, not\ 6$
10. Recurring Decimal	A decimal number that has **digits that repeat forever**. The part that repeats is usually shown by placing a dot above the digit that repeats, or	$\frac{1}{3} = 0.333 \ldots = 0.\dot{3}$ $\frac{1}{7} = 0.142857142857 \ldots = 0.\dot{1}4285\dot{7}$

	dots over the first and last digit of the repeating pattern.	$\frac{77}{600} = 0.128333\ldots = 0.128\dot{3}$

Topic: Factors and Multiples

Topic/Skill	Definition/Tips	Example
1. Multiple	The result of multiplying a number by an integer. The **times tables** of a number.	The first five multiples of 7 are: $7, 14, 21, 28, 35$
2. Factor	A number that **divides exactly** into another number without a remainder. It is useful to write factors in pairs	The factors of 18 are: $1, 2, 3, 6, 9, 18$ The factor pairs of 18 are: $1, 18$ $2, 9$ $3, 6$
3. Lowest Common Multiple (LCM)	The **smallest** number that is in the **times tables** of each of the numbers given.	The LCM of 3, 4 and 5 is 60 because it is the smallest number in the 3, 4 and 5 times tables.
4. Highest Common Factor (HCF)	The **biggest** number that **divides exactly** into two or more numbers.	The HCF of 6 and 9 is 3 because it is the biggest number that divides into 6 and 9 exactly.
5. Prime Number	A number with **exactly two factors**. A number that can only be divided by itself and one. The number **1 is not prime**, as it only has one factor, not two.	The first ten prime numbers are: $2, 3, 5, 7, 11, 13, 17, 19, 23, 29$
6. Prime Factor	A factor which is a prime number.	The prime factors of 18 are: $2, 3$
7. Product of Prime Factors	Finding out which **prime numbers multiply** together to make the **original** number. Use a **prime factor tree.** Also known as 'prime factorisation'.	

Topic: Accuracy

Topic/Skill	Definition/Tips	Example
1. Place Value	The **value** of where a **digit** is within a number.	In 726, the value of the 2 is 20, as it is in the ‘tens’ column.
2. Place Value Columns	The names of the columns that **determine the value of each digit**. The ‘ones’ column is also known as the ‘units’ column.	
3. Rounding	To make a number simpler but keep its value close to what it was. If the **digit to the right** of the rounding digit is **less than 5, round down**. If the **digit to the right** of the rounding digit is **5 or more, round up**.	74 rounded to the nearest ten is 70, because 74 is closer to 70 than 80. 152,879 rounded to the nearest thousand is 153,000.
4. Decimal Place	The **position** of a digit to the **right of a decimal point**.	In the number 0.372, the 7 is in the second decimal place. 0.372 rounded to two decimal places is 0.37, because the 2 tells us to round down. Careful with money - don’t write £27.4, instead write £27.40
5. Significant Figure	The significant figures of a number are the digits which **carry meaning** (ie. are significant) to the size of the number. The **first significant figure** of a number **cannot be zero**. In a number with a decimal, trailing zeros are not significant.	In the number 0.00821, the first significant figure is the 8. In the number 2.740, the 0 is not a significant figure. 0.00821 rounded to 2 significant figures is 0.0082. 19357 rounded to 3 significant figures is 19400. We need to include the two zeros at the end to keep the digits in the same place value columns.
6. Truncation	A method of approximating a decimal number by **dropping all decimal places** past a certain point **without rounding**.	3.14159265… can be truncated to 3.1415 (note that if it had been rounded, it would become 3.1416)
7. Error Interval	A **range of values** that a number could have taken before being rounded or truncated. An error interval is written using inequalities, with a **lower bound** and an **upper bound**.	0.6 has been rounded to 1 decimal place. The error interval is: $0.55 \leq x < 0.65$ The lower bound is 0.55 The upper bound is 0.65

	Note that the lower bound inequality can be 'equal to', but the upper bound cannot be 'equal to'.	
8. Estimate	To find something **close to the correct answer**.	An estimate for the height of a man is 1.8 metres.
9. Approximation	When using approximations to estimate the solution to a calculation, **round each number in the calculation to 1 significant figure**. $\approx$ means 'approximately equal to'	$\frac{348+692}{0.526} \approx \frac{300+700}{0.5} = 2000$ 'Note that dividing by 0.5 is the same as multiplying by 2'
10. Rational Number	A number of the form $\boldsymbol{\frac{p}{q}}$, where $\boldsymbol{p}$ **and** $\boldsymbol{q}$ **are integers** and $\boldsymbol{q \neq 0}$. A number that cannot be written in this form is called an 'irrational' number	$\frac{4}{9}, 6, -\frac{1}{3}, \sqrt{25}$ are examples of rational numbers. $\pi, \sqrt{2}$ are examples of an irrational numbers.
11. Surd	The **irrational number** that is a **root of a positive integer,** whose value cannot be determined exactly. Surds have **infinite non-recurring decimals**.	$\sqrt{2}$ is a surd because it is a root which cannot be determined exactly. $\sqrt{2} = 1.41421356 \ldots$ which never repeats.
12. Rules of Surds	$\boldsymbol{\sqrt{ab} = \sqrt{a} \times \sqrt{b}}$ $\boldsymbol{\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}}$ $\boldsymbol{a\sqrt{c} \pm b\sqrt{c} = (a \pm b)\sqrt{c}}$ $\boldsymbol{\sqrt{a} \times \sqrt{a} = a}$	$\sqrt{48} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$ $\sqrt{\frac{25}{36}} = \frac{\sqrt{25}}{\sqrt{36}} = \frac{5}{6}$ $2\sqrt{5} + 7\sqrt{5} = 9\sqrt{5}$ $\sqrt{7} \times \sqrt{7} = 7$
13. Rationalise a Denominator	The process of rewriting a fraction so that the **denominator contains only rational numbers**.	$\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{6}}{2}$ $\frac{6}{3+\sqrt{7}} = \frac{6(3-\sqrt{7})}{(3+\sqrt{7})(3-\sqrt{7})}$ $= \frac{18-6\sqrt{7}}{9-7}$ $= \frac{18-6\sqrt{7}}{2} = 9-3\sqrt{7}$

Topic: Fractions

Topic/Skill	Definition/Tips	Example
1. Fraction	A mathematical expression representing the **division** of one integer by another. Fractions are written as **two numbers separated by a horizontal line**.	$\frac{2}{7}$ is a 'proper' fraction. $\frac{9}{4}$ is an 'improper' or 'top-heavy' fraction.
2. Numerator	The **top** number of a fraction.	In the fraction $\frac{3}{5}$, 3 is the numerator.
3. Denominator	The **bottom** number of a fraction.	In the fraction $\frac{3}{5}$, 5 is the denominator.
4. Unit Fraction	A fraction where the **numerator is one** and the denominator is a positive integer.	$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\ etc.$ are examples of unit fractions.
5. Reciprocal	The reciprocal of a number is **1 divided by the number**. The reciprocal of x is $\frac{1}{x}$ **When we multiply a number by its reciprocal we get 1**. This is called the 'multiplicative inverse'.	The reciprocal of 5 is $\frac{1}{5}$ The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, because $$\frac{2}{3} \times \frac{3}{2} = 1$$
6. Mixed Number	A number formed of both an **integer part** and a **fraction part**.	$3\frac{2}{5}$ is an example of a mixed number.
7. Simplifying Fractions	**Divide the numerator and denominator by the highest common factor**.	$$\frac{20}{45} = \frac{4}{9}$$
8. Equivalent Fractions	Fractions which represent the **same value**.	$$\frac{2}{5} = \frac{4}{10} = \frac{20}{50} = \frac{60}{150}\ etc.$$
9. Comparing Fractions	To compare fractions, they each need to be rewritten so that they have a **common denominator**. **Ascending** means **smallest to biggest**. **Descending** means **biggest to smallest**.	Put in to ascending order : $\frac{3}{4}, \frac{2}{3}, \frac{5}{6}, \frac{1}{2}$. Equivalent: $\frac{9}{12}, \frac{8}{12}, \frac{10}{12}, \frac{6}{12}$ Correct order: $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$
10. Fraction of an Amount	**Divide** by the **bottom**, **times** by the **top**	Find $\frac{2}{5}$ of £60 $$60 \div 5 = 12$$ $$12 \times 2 = 24$$
11. Adding or Subtracting Fractions	Find the **LCM of the denominators** to find a common denominator. Use equivalent fractions to change each fraction to the **common denominator**.	$$\frac{2}{3} + \frac{4}{5}$$ Multiples of 3: 3, 6, 9, 12, **15**.. Multiples of 5: 5, 10, **15**.. LCM of 3 and 5 = 15

	Then just **add or subtract the numerators** and keep the **denominator the same**.	$\frac{2}{3}=\frac{10}{15}$ $\frac{4}{5}=\frac{12}{15}$ $\frac{10}{15}+\frac{12}{15}=\frac{22}{15}=1\frac{7}{15}$
12. Multiplying Fractions	**Multiply** the **numerators** together and **multiply** the **denominators** together.	$\frac{3}{8}\times\frac{2}{9}=\frac{6}{72}=\frac{1}{12}$
13. Dividing Fractions	**'Keep it, Flip it, Change it – KFC'** Keep the first fraction the same Flip the second fraction upside down Change the divide to a multiply Multiply by the reciprocal of the second fraction.	$\frac{3}{4}\div\frac{5}{6}=\frac{3}{4}\times\frac{6}{5}=\frac{18}{20}=\frac{9}{10}$

Topic: Basic Percentages

Topic/Skill	Definition/Tips	Example
1. Percentage	**Number of parts per 100.**	31% means $\frac{31}{100}$
2. Finding 10%	To find **10%**, **divide by 10**	10% of £36 = 36÷10=£3.60
3. Finding 1%	To find **1%**, **divide by 100**	1% of £8 = 8÷100 = £0.08
4. Percentage Change	$\boldsymbol{\frac{Difference}{Original} \times 100\%}$	A games console is bought for £200 and sold for £250. % change = $\frac{50}{200} \times 100 = 25\%$
5. Fractions to Decimals	**Divide the numerator by the denominator** using the bus stop method.	$\frac{3}{8} = 3 \div 8 = 0.375$
6. Decimals to Fractions	**Write as a fraction** over 10, 100 or 1000 and simplify.	$0.36 = \frac{36}{100} = \frac{9}{25}$
7. Percentages to Decimals	**Divide by 100**	$8\% = 8 \div 100 = 0.08$
8. Decimals to Percentages	**Multiply by 100**	$0.4 = 0.4 \times 100\% = 40\%$
9. Fractions to Percentages	Percentage is just a fraction out of 100. **Make the denominator 100 using equivalent fractions**. When the denominator doesn't go in to 100, use a calculator and **multiply the fraction by 100**.	$\frac{3}{25} = \frac{12}{100} = 12\%$ $\frac{9}{17} \times 100 = 52.9\%$
10. Percentages to Fractions	Percentage is just a fraction out of 100. **Write the percentage over 100** and simplify.	$14\% = \frac{14}{100} = \frac{7}{50}$

Topic: Calculating with Percentages

Topic/Skill	Definition/Tips	Example
1. Increase or Decrease by a Percentage	Non-calculator: **Find the percentage** and **add** or **subtract** it from the **original** amount. Calculator: Find the **percentage multiplier** and multiply.	<u>Increase 500 by 20% (Non Calc):</u> 10% of 500 = 50 so 20% of 500 = 100 500 + 100 = 600 <u>Decrease 800 by 17% (Calc):</u> 100%-17%=83% 83% ÷ 100 = 0.83 0.83 x 800 = 664
2. Percentage Multiplier	The **number** you **multiply** a quantity by to **increase or decrease** it by a **percentage**.	The multiplier for increasing by 12% is 1.12 The multiplier for decreasing by 12% is 0.88 The multiplier for increasing by 100% is 2.
3. Reverse Percentage	Find the **correct percentage given in the question**, then work backwards to **find 100%** Look out for words like '**before'** or '**original'**	A jumper was priced at £48.60 after a 10% reduction. Find its original price. 100% - 10% = 90% 90% = £48.60 1% = £0.54 100% = £54
4. Simple Interest	Interest calculated as a **percentage of the original** amount.	£1000 invested for 3 years at 10% simple interest. 10% of £1000 = £100 Interest = $3 \times £100 = £300$

Topic: Indices

Topic/Skill	**Definition/Tips**	**Example**
1. Square Number	The number you get when you **multiply a number by itself**.	**1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225…** $9^2 = 9 \times 9 = 81$
2. Square Root	The **number you multiply by itself** to get another number. The reverse process of squaring a number.	$\sqrt{36} = 6$ because $6 \times 6 = 36$
3. Solutions to $x^2 =$ ….	**Equations** involving **squares** have **two solutions**, one **positive** and one **negative**.	Solve $x^2 = 25$ $x = 5 \; or \; x = -5$ This can also be written as $x = \pm 5$
4. Cube Number	The number you get when you **multiply a number by itself and itself again**.	1, 8, 27, 64, 125… $2^3 = 2 \times 2 \times 2 = 8$
5. Cube Root	The **number you multiply by itself and itself again** to get another number. The reverse process of cubing a number.	$\sqrt[3]{125} = 5$ because $5 \times 5 \times 5 = 125$
6. Powers of…	The powers of a number are that **number raised to various powers**.	The powers of 3 are: $3^1 = 3$ $3^2 = 9$ $3^3 = 27$ $3^4 = 81$ etc.
7. Multiplication Index Law	When **multiplying** with the same base (number or letter), **add the powers**. $\boldsymbol{a^m \times a^n = a^{m+n}}$	$7^5 \times 7^3 = 7^8$ $a^{12} \times a = a^{13}$ $4x^5 \times 2x^8 = 8x^{13}$
8. Division Index Law	When **dividing** with the same base (number or letter), **subtract the powers**. $\boldsymbol{a^m \div a^n = a^{m-n}}$	$15^7 \div 15^4 = 15^3$ $x^9 \div x^2 = x^7$ $20a^{11} \div 5a^3 = 4a^8$
9. Brackets Index Laws	When raising a power to another power, multiply the powers together. $\boldsymbol{(a^m)^n = a^{mn}}$	$(y^2)^5 = y^{10}$ $(6^3)^4 = 6^{12}$ $(5x^6)^3 = 125x^{18}$
10. Notable Powers	$\boldsymbol{p = p^1}$ $\boldsymbol{p^0 = 1}$	$99999^0 = 1$
11. Negative Powers	A negative power performs the reciprocal. $\boldsymbol{a^{-m} = \frac{1}{a^m}}$	$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$
12. Fractional Powers	The denominator of a fractional power acts as a 'root'. The numerator of a fractional power acts as a normal power. $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$	$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$ $\left(\frac{25}{16}\right)^{\frac{3}{2}} = \left(\frac{\sqrt{25}}{\sqrt{16}}\right)^3 = \left(\frac{5}{4}\right)^3 = \frac{125}{64}$

Topic: Standard Form

Topic/Skill	Definition/Tips	Example
1. Standard Form	$\boldsymbol{A \times 10^{b}}$ $where\ \mathbf{1} \leq \boldsymbol{A} < \mathbf{10}, \quad \boldsymbol{b = integer}$	8400 = 8.4 x 10^3 0.00036 = 3.6 x 10^{-4}
2. Multiplying or Dividing with Standard Form	Multiply: **Multiply the numbers** and **add the powers**. Divide: **Divide the numbers** and **subtract the powers**.	$(1.2 \times 10^3) \times (4 \times 10^6) = 8.8 \times 10^9$ $(4.5 \times 10^5) \div (3 \times 10^2) = 1.5 \times 10^3$
3. Adding or Subtracting with Standard Form	**Convert** in to **ordinary** numbers, **calculate** and then **convert back** in to standard form	$2.7 \times 10^4 + 4.6 \times 10^3$ $= 27000 + 4600 = 31600$ $= 3.16 \times 10^4$